# A study of the socio-economical impact of Mathematics in France

Mathematics, an essential asset for addressing tomorrow's challenges: knowledge, innovation and competitiveness.

**Synthesis**

DÉNOUER LES PROBLÉMATIQUES COMPLEXES EST UN ART

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## INTRODUCTION

**Although the excellence of French research in mathematics is world renowned, its interaction with the industrial world still lacks clarity**.

**Yet, industry and the business world abound in mathematics** more or less explicitly (models, statistics, algorithms, numerical simulation, optimization, etc.).

> *"The domains that mobilize advanced mathematics are today considerably more numerous than twenty years ago. They are also more strategic. (...) Today we see new professions appearing and new economical models, in which statistics and data analysis play a major role. The collection, structuring, transformation and exploitation of collected data traverse high-level mathematical processes. (…) In the new paradigm, marked by the continuity between fundamental and applied mathematics and by the presence of fundamental mathematics at the heart of the economical world, the question of communication is central."*
>
> *Jean-Pierre Bourguignon, President of the European Research Council, « Un nouvel âge d'or pour les Mathématiques en entreprise ? » (2014)*

This tendency has been confirmed elsewhere by an ensemble of reflections led over the past few years by mathematics communities all over the world, notably in France at the behest of learned societies[1], but also in Europe[2] and in the United States[3], as well as in Australia[4] and Canada[5].

Recently, reports have noted highly significant impacts of mathematics in terms of employment and added value. A study on the United Kingdom[6] estimated at 2.8 million the number of jobs that use mathematical competencies, namely 9% of total employment, and at 208 billion pounds their contribution to added value, that is 16% of Gross National Product (GNP).

In this context, the Agency for Interactions of Mathematics with Enterprises and Society (AMIES), in partnership with the Paris Foundation of Mathematical Sciences (FSMP) and the Jacques Hadamard Mathematical Foundation (FMJH) in association with mathematics labs of excellence (Labex), entrusted to the strategic advisory cabinet CMI **a study of the socio-economical impact of mathematics in France**. This study, which was conducted from January to May 2015, has three major objectives:

- Measure the socio-economical impact of mathematics in France and demonstrate the actual economical contribution of mathematics to industrial development and to innovation – socio-economical impact seen here as both the direct impact (volume of jobs, proportion of sectorial and global GNP) and indirect (major technological developments facilitated by being based on mathematical competencies);

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[1] ARP, Enquête sur les mathématiques au cœur de l'innovation industrielle (2008), SMAI - Rapport de prospective sur les mathématiques appliquées et industrielles (2008)
[2] ESF, Forward Look on Mathematics and Industry (2011)
[3] SIAM, Report on Mathematics in Industry (2012)
[4] Australian Academy of Science, The importance of advanced physical and mathematical sciences to the Australian economy (2015)
[5] Some Assembly Required : STEM Skills and Canada's Economic Productivity, The Expert Panel on STEM Skills for the Future, Council of Canadan Academies, 2015
[6] Deloitte, Measuring the Economic Benefits of Mathematical Science Research in the UK (2010)

- Analyse the relationships between businesses and expertise in mathematics to enlighten directors of research program and management in businesses for their development strategies and the integration of mathematical competencies;
- Specify the thematic domains as well as the tools that should be developed for a better synergy between mathematics and businesses.

The results of this study report the **particularly diffusing character of mathematics**; mathematics constitutes a field of research, of learning and of knowledge that serves as a basis for the development of fundamental competencies, mobilized in a large number of professions and of leading competencies in advanced[7] mathematics for solving more and more complex problems.

Today we remark a great transversality of the contribution of mathematics, as well as its growing importance for solving industrial problematics. We can cite:

- The need to apprehend complex systems, indeed systems of systems that must interact among themselves;
- The need to reason on several distinct scales (eg. Molecular scale to systemic scale);
- The management of uncertainties.

Nevertheless, **all the problematics for which mathematics are mobilized are not solved by mathematics alone**. The results of this study underline the very strong interactions, that have a tendency to accentuate, between mathematics and other disciplines: computer science, physics, mechanics, system control, chemistry, life sciences, social sciences, etc. Close links are established between progress in mathematics and progress in the other academic fields and it is these interactions between evolving disciplines that make mathematics a *key enabling technology*.

If we refer to the definition given by the European Commission of key enabling technologies, mathematics, in interaction with other disciplines, seems in fact to present all the necessary characteristics:

- *It is knowledge- and investment intensive.*
- *It is based on a high level of research and development.*
- *It generates rapid and integrated cycles of innovation.*
- *It generates highly qualified employment.*

*Its influence is omnipresent, enabling innovations in processes, in products and services within the entire economy. It falls within a systemic approach,* which is *pluri-disciplinary and trans-sectorial, diffusing into numerous technological domains.*

The ensemble of these results is detailed in the present document, which is articulated in the following manner:

[7] Here, advanced mathematics are taken as mathematics that mobilize research in mathematics and the tools produced by this research.

- The first part, “Panorama of research and training in mathematics”, delimits the perimeter of mathematics in research, describes its relationships with the socio-economical world and clarifies the place of mathematics in education.
- The second part, “Quantification of the socio-economical impacts of mathematics”, draws up the list of professions that mobilize competencies in mathematics, as well as its weight in the French economy in order to deduce the impact of mathematics in terms of employment and the added value by sector and to the French Gross National Product.
- The third part, “Contributions and diffusion of mathematics”, displays the competencies contributed by mathematics, which are strategic for business development, the vectors of diffusion and mode of integration.

The methodology is based on an exhaustive documentary analysis, the conducting of interviews among more than forty researchers, industrialists and experts and data analysis of statistics produced by INSEE[8], for quantifying the socio-economical impact of mathematics. On this point, we identified the professions directly impacted by mathematics (training in or by mathematics, production and application of tools or research in mathematics), we attributed these professions to the different economical sectors and we applied an Input-Output model for computing the employment and the added value attributable to the mobilization of mathematics.

[8] Data of 2012. The attribution of professions impacted by mathematics the different economical sectors was done by intersecting the 415 posts/professions listed by INSEE within their Socio-professional Professions and Categories (PCS) reference with the 615 activity sectors listed in the French Activities Nomenclature (NAF)

## SUMMARY

This study emphasizes the very strong and growing impact of mathematics for the competitiveness and growth of the French economy:

- *The jobs affected by mathematics have a strong added value (15% of GNP and 9% of employment) and are increasing in number (+0.9% per year from 2009 to 2012 vs. +0.5% for overall employment)*
- *It turns out that 44% of key technologies, identified as such by government reports, are strongly affected by progress in mathematics.*
- *The mobilization of 5 major competency fields of mathematics (signal and image analysis, data mining, modelling-simulation-optimization (MSO), high performance computing (HPC), computer system security and cryptography) will increase in numerous activity sectors, in particular energy, health, and telecommunications.*

The employability of mathematics students is excellent: businesses (both large ones and SME's) are progressively becoming aware of this impact but are still somewhat ill-organized for managing in-house mathematical expertise.

The study finally underlines the necessity to reinforce the links between the higher education system and the businesses, in particular for the universities:

- *Clarity of the higher education and research system is still too weak.*
- *Attractiveness of careers for PhD's in businesses is still insufficient.*
- *Initiatives of support for mathematical expertise in SME's still need reinforcing.*

By seeking to correct these points of weakness, the scientific excellence of French mathematics will truly constitute a competitive advantage for the economy.

## SYNTHESIS OF THE STUDY

### *A PANORAMA OF RESEARCH AND TRAINING IN MATHEMATICS: QUALITY AND DIFFUSION OF MATHEMATICS AS AN ACADEMIC DISCIPLINE*

**French research in fundamental and applied mathematics is world-renowned, but still indirectly represented in major national and European programs.**

French public research is at the forefront in a large number of disciplines, both fundamental and applied. Today, there are 4 000 researchers and lecturer-researchers[9] in mathematics within more than 60 national laboratories, whose work is internationally recognized. With 8,5% of the most cited publications in mathematics at 2 years[10] in the world in 2012, France is placed behind the United States (27,3%) and Chine (20,5%) but ahead of Germany, Italy and the united Kingdom (between 5,8% and 7,6%). Mathematics appears in 3rd place among the French disciplines for their place in mathematics publications most cited at 2 years in the world (top 10%), behind Planetary Sciences (11,2%) and Physics (9%).

**Mathematics is nonetheless indirectly represented in major national and European programs.**

While the *European Science Foundation* underlined in 2011 in its *Forward Look "Mathematics and Industry"* that research in mathematics was severely under-represented within major European funding programs (H2020 program), mathematics is more present since 2014, notably through HPC, featuring in several calls. In spite of this, mathematics as such is not represented. The same goes for national calls, such as those of the Agence Nationale de la Recherche (except for the call « Défis de tous les savoirs »). It is in the interaction with other disciplines (Biology Health, Energy, Engineering, ICT) that financing opportunities for mathematics appear.

Conversely, the *Division of Mathematical Sciences* (DMS) of the US *National Science Foundation (NSF)* directly targets programs and financing opportunities at mathematics.

**A wide diversity in the modes and sectors of socio-economical collaborations, but contractual research-industry relationships that remain, still not sufficiently structured.**

The collaborations and the relationships between public research and the socio-economical world operate according to very varied modes – creation of start-ups, contractual and joint research, joint labs, industrial chairs, etc. – and we estimate that approximately 10% of researchers in mathematics maintain regular relationships with businesses. However, direct relationships with industry, in the framework of contractual collaborations, still remain almost insignificant. We estimate that the proportion of private contractual resources in the consolidated budget of the average laboratory is between 1% and 5%. By comparison, we estimate at 4%[11] the proportion of research contracts signed with businesses for all the resources of public research operators. Even if several mathematics research labs exceed this level, the majority are below it.

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9 Numbers principally from INSMI and other CNRS Institutes, from INRIA and from French universities.
10 Even though this metric is not favourable to the measurement of the quality of mathematics production, combined with other metrics related to the level of internationalization of publications, to the number of medals awarded to the French community, it allows one to position France among the international leaders of the discipline.
11 Law of Finance 2015. Extract from the budget "blue" for Research and Higher Education.

The research-business relationship still lacks structuring; the labs have no or few human resources dedicated to running and maintaining relationships with businesses; the academic community is traditionally not orientated towards industry; the collaborations with the private sector are also hampered by the difficulties that SME's have in mathematically formulating their problematics; the very constrained nature of calls for collaborative projects by ANR, H2020 or FUI undermine this relationship; the academic, technological and instrumental skills of the mathematics academic community lack clarity at a national level.

Levers for reinforcing the research-industry relationship related notably to community networking at a national level, to setting up structuring partnerships with large groups or to development of approaches for enhancing the transfer adapted to SME's could be more actively mobilized.

On this last point, numerous high-visibility initiatives - the project hotel Maimosine in Grenoble, HPC-PME around INRIA, GENCI, BPI France and CEMOSIS in Strasbourg - aim in particular at reinforcing the research-industry relationship at the scale of localized and well-identified communities. The key success factors of these initiatives lie in the clear posting of mobilizable competencies, in most cases through a pluri-disciplinary approach (mathematics, computer science, industrial engineering, system control, etc.), in the formal accompanying of the business aimed at allowing the translation of an industrial project's needs into the relevant need for mathematical competencies, in the quality of the support by « project engineering » all the way up to its financial engineering, and in the setting up of relationships with the pertinent scientific experts.

Beyond this, events and initiatives are also organised for encouraging research-industry relationship by AMIES, the Laboratories of Excellence in Mathematics, the learned societies, the FSMP… This dynamic must be pursued.

**The diffusion of mathematics in education itself is broad and provides the competencies expected by the professional world.**

**The student numbers (Master and Doctorate) in mathematics are stable in a training discipline de formation that weighs relatively little**: owe estimate at 2,1% the proportion of Master's students for whom Mathematics is their major. There were 6 600 enrolled in 2012-2013. 43 doctoral schools are authorized to award a Doctorate in Mathematics[12]. In 2012-2013, there were close to 2 000 registered, that is 2,9% of the numbers in all doctoral training.

**Mathematics has, on the other hand, a significant weight in other training programs at levels Bac+2 to Bac+8, training <u>by</u> mathematics[13] is considerable.** To the 6 600 Masters students in Mathematics we can add about 7 780 students trained by mathematics, registered in one of the 145 university Masters programs that strongly mobilize mathematics. Furthermore, 56% of students in DUT, 10.4% of students in BTS, 1.2% of students in Professional undergraduate programs are trained by mathematics.

As far as engineering degrees are concerned, among all the courses taught, mathematics counts for approximately 16% of first year courses, 10% of second year courses and 6% of third year courses.

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12 Option « Mathematics and their interactions »

13 Training by mathematics implies here that the programs mobilize mathematics courses significantly (more than 4h per week in average throughout the post baccalaureate training program)

Globally, **we can thus estimate at 25% the proportion of student numbers registered in BTS, DUT, professional undergraduate, engineering schools, Master or Doctorate** who are trained in mathematics and by mathematics (this represents approximately 2.1 million working people, trained in or by mathematics over the last 35 years).

These degrees have excellent professional insertion.

**The prospects in the private sector for students trained in Mathematics are principally beneficial for Masters students.**

The Masters graduates in mathematics mostly work in the private sector whereas the opposite tendency holds for PhD's (75% of them work in public research and education[14]). This raises the question of the attractiveness of private careers for PhD's in Mathematics but also that of prospects for PhD's in the private sector.

## *SOCIO-ECONOMCAL IMPACTS OF MATHEMATICS: WEIGHT OF MATHEMATICS IN EMPLOYMENT AND FRENCH GNP*

**Mathematics in France directly impacts 9% of jobs in 2012. This represents about 2.4 million jobs. The added value contributed by mathematics in France represents 285 billion euros out of 1 878 billion euros [15], that is 15% of GNP.**

**The number of posts[16] directly impacted by mathematics in France amounts to 3.8 million out of 43.3 million that is 9% of the total number of jobs.**[17], all sectors of activity included.

This places France with **2.4 million jobs impacted by mathematics** on a comparable level to the United Kingdom that has 2.8 million, that is **10% of total employment in 2010** (an equivalent study in the Netherlands shows 11% of jobs impacted by mathematics).

This weight is in constant progression since 2009: the number of posts directly impacted by mathematics has increased faster than the total number of posts in France over the period (0.9% in average annual growth rate over the period 2009-2012 for posts impacted by mathematics vs. 0.5% for the totality of posts).

The **three regions that contribute the most – Ile-de-France, Rhône-Alpes and Provence-Alpes-Côte d'Azur -, all jobs considered, regroup 56% of jobs impacted by mathematics**. The accumulation of employment impacted by mathematics in these regions is greater than the accumulation of total employment (42.3%). This is in particulrr the case for Ile-de-France, who concentres 38.4% of mathematical employment vs. 24.5% of total employment.

The **added value provided by mathematics in France** represents **285 billion euros** out of 1 878 billion euros[18]. **This represents 15% of GNP**. This weight is in **constant progression since 2009**. It is slightly inferior to that observed in the United Kingdom, that comes to16% (278 Billion€ compared

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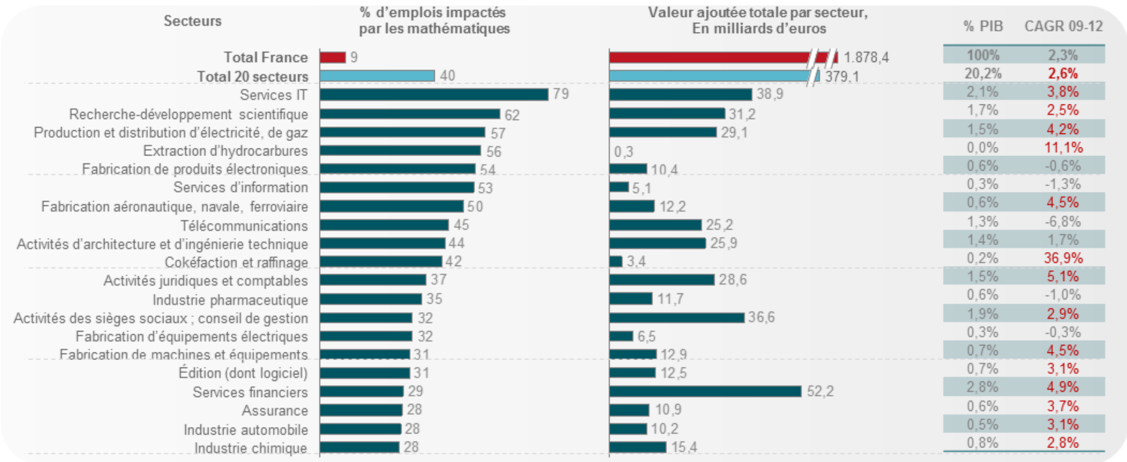
[14] Estimation made from a study on professional insertion of mathematics PhD's in Ile-de-France: AMIES, La poursuite de carrière des docteurs récemment diplômés en IDF, Résultats pour les docteurs en Mathématiques (2015)
[15] without VAT, customs and taxes, subventions
[16] The study of CMI, based on the same approach as that of Deloitte in their UK study, privileges an analysis of the number of posts impacted by mathematics as input data (vs. Deloitte in number of salaried employees). A post corresponds to a profession occupied by a salaried employee in an establishment. The number of posts is then translated into number of jobs.
[17] Latest data available from INSEE (2012)
[18] without VAT, customs and taxes, subventions

with 1 807 Billion€ in 2010) but superior to the weight of mathematics in the Dutch GNP, estimated at 13%. The differences in the weight of mathematics for employment and the GNP between France and the United Kingdom can essentially be explained by the structure of the Gross National Product of each country. Two of the sectors that are most impacted by mathematics weigh more in the British GNP than in the French: the sector Information and Communication that weighs 4.5% in the French GNP, 5.5% in the United Kingdom GNP and the Finance and Insurance sector that weighs 3.8% in the French GNP against 7.7% in the United Kingdom GNP.[19]

**The 20 sectors that contribute mostly in terms of employment impacted by mathematics that increase GNP growth: their growth, between 2009 and 2012, is 2.6% per year on average against 2.3% for the ensemble of sectors over the same period. These sectors represent 70% of jobs impacted by mathematics (vs. 50% of total jobs).**

More specifically, 15 of the 20 sectors that contribute the most, have growth significantly superior to GNP. This concerns notably IT Services, energy (electricity, hydrocarbons, etc.), industry (automobile, aeronautics, railways, etc.), banking, finance and insurance.

**Graphic**: The top 20 sectors the most impacted by mathematics in terms of employment (2012)




Among these sectors, the professions the most impacted by mathematics are:

- The engineers: IT (including software, etc.) principally, construction, agri-food, technico-commercial (machines, etc.)
- The executives of financial services.
- The management staff of the public sector.
- The engineers and technicians (with a qualification in mathematics) for part-time related to employment.

**It is estimated that 30% of jobs mobilizing mathematics arise from the direct application of research results and mathematical tools, apart from the informatics and financial sectors.**

The professions directly involved in the production and the application of research in sciences mathematics and mathematical tools represents 1.1 million posts, namely close to 30% of posts

[19] Comparisons made for the years of reference of both studies: 2010 for the Deloitte study in the UK, 2012 for France in the present study.

impacted by mathematics and 2.6% of total employment. These professions boost the growth in the number of jobs directly impacted by mathematics, with an average annual growth rate of +1.5% between 2009 and 2012.

The top 5 sectors that contribute the most, in order are:

- Teaching.
- Scientific research-development.
- Architectural and engineering activities; control and technical analysis activities.
- Specialized construction work and manufacturing of means of transport.

It is in these sectors that the professions of Lecturers and Lecturer-Researchers, Engineers (outside of the informatics sector), R&D Engineers, R&D Technicians, are concentrated. They generate **79,8 billion euros of added value**, namely close to **4.3% of total GNP**.

## *CONTRIBUTION AND DIFFUSION OF MATHEMATICS: WHAT IMPACTS ON THE DEVELOPMENT OF BUSINESSES, WHICH MODES OF INTEGRATION FOR MATHEMATICAL COMPETENCIES?*

**Mathematics is an essential factor in the creation of value and contributes significantly to the development of future technologies.**

**Advanced mathematics**, mobilized notably in the above-mentioned sectors, **provide incomparable tools** based on computation, statistics and probability. It offers a coherent logical framework and a universal language for the analysis, the simulation, the optimization and the control of industrial processes.

Coupled with computer science, and in interaction with the science(s) of the applied sector concerned, mathematics enable the construction and the manipulation of complex models and proposes numerical simulations at the origin of value creation in industry and services.

Beyond the growth that it generates, advanced mathematics contribute to French competitiveness. In the sectors that mobilize it, mathematics is essential for the development of 44% of key technologies[20] recognized as strategic levers for the competitiveness of businesses: **out of the 85 technologies listed, 37 see their progress significantly conditioned on the progress in the mathematics domain.**

**An essential role for mathematics in industrial development, called upon to be reinforced via the mastery by businesses of all or some of the 5 strategic competency fields ranging from fundamental to applied mathematics.**

The contribution, invisible, direct or indirect, but primordial of mathematics in the development of key technologies and/or products, processes and services that transform day-to-day life, should play in favour of a broad recognition of its strategic interest at a European level, probably at the same level as the six key enabling technologies (nanotechnologies, microelectronics, biotechnology, photonics, advanced materials, advanced production / manufacturing systems) targeted in the framework program Horizon 2020.

This contribution appears notably thanks to the mobilization, in several activity sectors, of 5 major fields of strategic competencies, founded, wholly or partially[21], on mathematics:

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[20] Ministry of Industry, Prospective Study « Technologies clés à l'horizon 2015 » (2010)

[21] These fields of competencies mobilize also computer science and physics and other scientific domains related to their fields of applications (biology, chemistry, etc.)

- **Signal processing and image analysis.**
- ***Data Mining*** (statistics, data analysis and machine learning)
- **MSO** (Modelling - Simulation – Optimization)
- **HPC** (High Performance Computing)
- **Security of IT systems and Cryptography**

The mastery of these competency fields by businesses is considered as essential for enabling them to take up the actual and future industrial challenges, specific or not to their activity sector, and to remain competitive.

**Several modes of integration for competencies present over the entire value chain of businesses, whose mobilization should increase in the coming years.**

**The modes of integration of mathematical competencies in the organisation of businesses** and notably those related to the above 5 fields vary principally as a function of their R&D strategies and their development priorities. Two complementary approaches are conceivable. They can choose to internalize the mathematical competencies that are strategic for their competitiveness via the recruitment of "mathematicians" or the investment in tools having a rich mathematical content for strategic (and confidential) activities and projects. They can also turn to academic research to test hypotheses and study new fields of development in more prospective subjects, potentially generators of income, but on a longer term or even solicit start-ups specialised in algorithmics, in modelling et numerical simulation…

The internalizing of mathematical within businesses can be done in various manners: targeted recruitment of PhD's or engineers with PhD's in mathematics, experts in the domain; recruitment of engineers endowed with skills in advanced mathematics, capable of understanding the models developed by experts and of using them in a pertinent way; use of tools that are based on and include advanced mathematics.

**Mathematical competencies are mobilized over the entire value chain** – from R&D to commercialization passing through production – from the most basic level (proportions, percentages, use of spread sheets, etc.) to the most advanced level according to the speciality (statistics for reliability computations, simulation-prediction in industrial processes, etc.). This mobilization could increase in the coming years. For example, in « Un nouvel âge d'or pour les Mathématiques en entreprise ? », Jean-Pierre Bourguignon refers to Veolia who plans to increase the proportion of "mathematicians" in its engineering workforce from 8% to 20% over a period of ten years.

We observe moreover an increase in the number of targeted recruitments, in particular with a profile of ***data analytics***. Hence we remark the emergence of recruitment campaigns of "mathematicians", for the new profession of "*data scientists*" or the establishment of new teams in statistics and modelling, in bio-statistics.

But the translation of this need, for *data analytics* in particular, not only by businesses, in terms of expected competencies, but also by the academic community, in terms of training programs, has not completely occurred.

**A diversity of mathematical profiles sought but an ideal type: the engineer-PhD, in particular within the large groups who's R&D is very strongly linked to industrial operations.**

The most important aspect seems to be the combination of **specialist expertise** and the capacity **of investigation at the interfaces**. The distinction is not necessarily made at the training levels themselves. It is done at the level of the skills developed such as autonomy, competency in project management, capacity to train and work in a team …

This is particularly the case in large groups. The characteristic is reinforced in businesses where R&D and industrial operations are tightly linked, the R&D engineers intervening notably directly in the design of industrial projects and the manufacturing.

The tendency differs in SME's: paradoxically, though SME's enter less easily into contractual research relationships and they are more distant from academic laboratories than the large groups, they call upon university profiles more easily, integrated in the business in the framework of initial employment (notably post-docs) or in the framework of a CIFRE contract on targeted projects.

**A contribution and a potential that are increasingly objectivized and generate more offensive and structured integration strategies.**

If the targeted recruitment of personnel for their "expertise" in mathematics ("senior expert in mathematics", "expert in algorithmics", "expert in numerical simulation"), constitutes today rather an exception than the rule, the perspectives of integration of expert competencies in *data analytics* and the adaptation to the digital revolution, within numerous sectors, could bring about an evolution of this situation.

Coupled to the issues of more and more complex simulation and optimization in the domains of Energy (multi-scale and multi-time treatment), of Communication (management of chain reactions in networks), of Health (simulation of functioning of organ and brain systems), of Industry in design, production, maintenance (management of batch production), of Services (steering of multi-scale logistic networks)…, the integration of these competencies represents a considerable potential and enterprises are starting to develop integration strategies that are more offensive and structured, aimed at recruiting the best profiles wherever they come from, in France or overseas, from Engineering Schools or from Universities, in particular for reinforcing their R&D teams and departments.

We can thus consider that the observation in the report of the *European Science Foundation*, *Forward Look "Mathematics and Industry"* in 2011, according to which mathematics is characterized by "an invisible contribution with visible successes", must be nuanced. The actual or potential contribution of Mathematics is more and more borne out in businesses that accurately measure gains in productivity related to the integration, already quite massive, of competencies in MSO and that evaluate the potential of integration of new competencies. The testimony of the Safran group supports this: "The experience of implementing a PHM (Prognostic & Health Monitoring) system for the motors having been a success, conscious of the opportunities that data analysis offers, we have decided to analyse other sources, in addition to operational data recorded during flights; such as those issued from production and test of motors. Once again, '*datalabs'* could be envisaged within the Safran Group to participate in efforts aux efforts hoped for with the introduction of the unit Safran-Analytics." (Safran)

**This supposes, both from businesses and the academic community, the deployment of approaches for improving status and follow-up of competencies.**

With the exception of a few large groups who have a career stream in expertise (parallel to that of management) and who on this basis "label" the mathematical competencies (algorithmics, statistics, system control), rare still are the personnel recruited for their mathematical competencies who are identified, classed, counted and followed as specialists in the field of mathematics.

Moreover, in the majority of contexts, the ***mathematical competencies***, if they remain useful, are less and less used during the evolution of the career of mathematicians: they turn to managerial functions or evolve towards posts of project directors, in R&D, losing the daily contact with mathematics.

This interrogates the **capacity of businesses and their personnel to conserve a sufficient level of expertise in mathematics**, to renew their competencies, to identify within the business or outside of it the pertinent resources for playing a part in new programs or new projects.

With the growing expression of specific needs (*data analytics*, HPC, etc.), the question of the capacity to rapidly identify the expert, hybrid or transverse competencies and not specifically related to the object of studies led by young graduates (for example, competencies in MSO are likely to be acquired in the framework of mathematical training, but also in physics training mobilizing computation and simulation…) will be asked or is already being asked.

The academic community initiated, in this perspective, the creation of a Label C3I to attest to competency in HPC of PhD's. To date, one hundred PhD's have been awarded this label. The principle could be to extend this to fields of MSO or even *data analytics* or "big data" in the future.

## *KEY FIGURES*

**4 000** researchers and lecturer-researchers

**500** PhD's per year

**60 principal research labs of which 42** labs of **INSMI**

**25%** of students at levels between Bac+2 and Bac+8 are trained in or by mathematics

**2.1 million** trained students employed in 2015

**8,5%** of the workforce

**37 key technologies out of 85** are impacted by mathematics

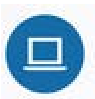
**Of which 11 are very strongly impacted**

- Molecular Simulation
- Nuclear energy
- Smart grids
- Exploration and production of hydrocarbons.
- Genetic engineering
- High performance computing
- Technologies for life-imaging
- Complex systems engineering and systems of systems
- Progressive/Intelligent Manufacturing
- Holistic security
- Communications and data

**3.8 million** posts impacted by mathematics, **2.4 million jobs, that is 9%** of employment

**285 Billion€** of added value
**15%** of GNP

**Top 5 sectors most impacted by mathematics (weight of employment related to mathematics by sector)**



IT Services: **75%**

Scientific R&D: **62%**

Production and distribution of electricity and gas: **57%**

Extraction of hydrocarbons: **56%**

Producation of electronic products: **54%**

**56% of employment impacted by mathematics is concentrated in 3 regions: Ile-de-France, Rhône-Alpes and Provence-Alpes-Côte d'Azur**

**15 sectors among the top 20 of sectors the most impacted by mathematics exhibit growth rates superior to that of the French GNP.**

## *CONCLUSIONS AND PERSPECTIVES FOR ADDRESSING TOMORROW'S CHALLENGES: KNOWLEDGE, INNOVATION AND COMPETITIVIENESS*

In spite of a relatively modest workforce (4000 researchers and lecturer-researchers), **French research in Mathematics holds a world-leading position**. The French scientific system in mathematics is also particularly well-connected at an international level (close to half of the publications are co-authored in the framework of international collaborations, far ahead of the US or China).

**This situation potentially constitutes a significant competitive advantage for the French economy**, due to the considerable crosscutting nature of the discipline and the crucial issues that are addressed through its contribution to 5 key domains: signal and image analysis, data mining, modelling-simulation-optimization (MSO), high performance computing (HPC) and issues related to computer system security; progress in these domains will condition our competitiveness in sectors as strategic as energy, health, banking and insurance as well as telecommunications.

**This study demonstrates the strong socio-economical impact of mathematics in France, compared to our neighbours**: 9 % of jobs impacted (versus 10% for the United Kingdom, 11% for the Netherlands), jobs with strong added value (15% of French GNP versus 16% for the United Kingdom), jobs bringing more growth (+0,9 % for jobs strongly impacted by mathematics versus 0,5% on average in France).

The interviews that were conducted in companies reveal both a **growing need for mathematicians** and an increasing awareness of the issue of well integrating mathematicians.

**This overall diagnostic is very favourable for mathematics. However, it encounters several weaknesses:**

- *Structuring in the research-industry relationship.*
- *Clarity of the higher education and research system.*
- *Attractiveness of study courses in mathematics (stability of registrations) and career pathways in industry for PhD's even though the professional perspectives are excellent for those that have followed these courses.*
- *Follow-up of mathematical careers in companies.*

In this context, it seems that four major dimensions should be taken into account in order to address these challenges of tomorrow in the fields of knowledge, innovation and competitiveness based on mathematics.

**1. The quality of research-industry relations, by reinforcing the capacities and levers for contractual research, joint research and transfer.**

The relationship research-industry, both with SME's and large groups, deserves to be accompanied. Several orientations can be considered:

- The extension of initiatives such as the project hotels **Maimosine, HPC-PME or CEMOSIS** that seem to have proven their added value in their relationships with SME's. This type of initiative could be deployed in the optic of a more structured increase of value of skill offers and favouring of contractual research.
- The stressing of the participation of mathematics research labs in **Carnot Institutes**, for accelerating the development of contractual and joint research, with SME's and large groups. This participation relies strongly on the perpetuity of existing tools, in particular for SME's (access conditions to tools, level of co-financing for collaborative projects, mobilization of Research Tax Deductions).
- The continuation of **research-industry encounters, thematic days or seminars,** etc. organized notably by AMIES, Laboratories and Laboratories of excellence in Mathematics, learned societies, or even the FSMP, coupled with the reinforcing of the national network of industrial correspondents within the mathematics labs by additional human resources, mutualized or not;
- The increased implication of mathematics communities in their **regional innovation ecosystems** (competitiveness Poles, SATT) for favouring this dialog and the opening of the mathematical community towards broader application domains;
- The adaptation of **evaluation criteria for research careers** (CNU committees 25 and 26), by integrating criteria related to taking into account research activities applied to transfer.

**2. The capacity to adapt training paths and enhance the status of the acquired skills.**

This dimension seems particularly strategic for answering the needs of integration in businesses specific competencies and facilitating the increased prestige of competencies acquired by young graduates (Master, Engineers, PhD's in particular) and their professional insertion. Several levers could be mobilized.

- **The conducting of a reflection organised in collaboration with industrials on the evolution if academic and professional competencies expected, in particular in the domain of *data analytics and HP*;**
- **The extension of labelling campaigns of competencies acquired for PhD's** in these same domains and in those of MSO while ensuring a proper recognition, by businesses, of the labels concerned;
- **The enhanced opening of training in mathematics towards industry and other disciplines:** opening of training in mathematics for thematics arising from application domains, increasing industrial encounters in the framework of doctoral training, development of targeted mechanisms for transition of doctoral students to businesses during their training to promote public-private mobility (CIFRE thesis of mathematicians are rare, and the doctorate-consultant is almost unknown, though this mechanism exists since 2009.)

**3. The continued mobilization of mathematical competencies by industry, their follow-up and the enhancement of the value of mathematical careers in the private sector.**

The added value of the mobilization of competencies mathematical tools, be they advanced or not, seems established for a large number of sectors and will get stronger in the coming years. It is certainly within the **SME's the least active in R&D** that this integration is the least. A majority of the above-mentioned levers could facilitate a **wider diffusion of mathematical competencies and tools in these businesses**.

Beyond this, it seems to us that an effort still needs to be made in business to **ensure the follow-up of personnel considered strategic for their mathematical competencies,** updating the competencies of their mathematicians…, and allowing them to conserve a sufficient expertise.

Finally, the attractiveness of scientific careers remains a national problem - the domain of mathematics is also affected -, it seems to be critical to **reinforce the added prestige of professional prospects associated to training paths in mathematics**.

**4. The continued support of research and training in mathematics.**

The preservation, even the increase of the socio-economical impact of mathematics will have to go through **a stronger exposition to the mathematical community** and the **reinforcing of the clarity of French competencies** both in research and in education. The reinforcing and the structuring of public relations at a national level of French mathematical competencies (research and training) and their applications, extended to contents communicated by INSMI, AMIES, the learned societies, the FSMP, the IHP to offers competencies, to large domains of competencies in training, etc. could be envisaged. These public relations could be part of a global approach, including also secondary education.

The access to **points of contacts per laboratory** (with clearly identified representatives) for the research-industry relationship could be strengthened.

Beyond this, **participation in a national and European lobbying effort for promoting mathematics** as a key technology and ensuring **a more direct representation within major national and international programs** (through the MENESR, AMIES, INSMI, the learned Societies, EU-MATHS-IN, ECMI, EMS, …) is an important lever for adding prestige to French competencies in mathematics and its contribution to economical development and to innovation.

Finally, the **continuation of direct support to French research** seems today to be strategic for allowing France to conserve its position in the international panorama, in particular for the fields of research that contribute to the 5 major competency domains mentioned on page 14.