Ejemplos internacionales de ideales de docencia en matemáticas

Enrique Acosta
Sociedad Colombiana de Matemáticas
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Chile (2012)

Estándares orientadores para carreras de pedagogía en educación media.
Ministerio de Educación

"…los estándares presentados en este documento orientan los conocimientos y habilidades que debe demostrar el futuro profesor o profesora de Educación Media para desempeñarse en los seis grados que comprende este nivel de escolaridad. "

ESTÁNDARES PEDAGÓGICOS

Estándar 1:	Conoce a los estudiantes de Educación Media y sabe cómo aprenden.
Estándar 2:	Está preparado para promover el desarrollo personal y social de los estudiantes.
Estándar 3:	Conoce el currículo de Educación Media y usa sus diversos instrumentos curriculares para analizar y formular propuestas pedagógicas y evaluativas.
Estándar 4:	Sabe cómo diseñar e implementar estrategias de enseñanza-aprendizaje adecuadas para los objetivos de aprendizaje y de acuerdo al contexto.
Estándar 5:	Está preparado para gestionar la clase y crear un ambiente apropiado para el aprendizaje según contextos.
Estándar 6:	Conoce y sabe aplicar métodos de evaluación para observar el progreso de los estudiantes y sabe usar los resultados para retroalimentar el aprendizaje y la práctica pedagógica.
Estándar 7:	Conoce cómo se genera y transforma la cultura escolar.
Estándar 8:	Está preparado para atender la diversidad y promover la integración en el aula.
Estándar 9:	Se comunica oralmente y por escrito de forma efectiva en diversas situaciones asociadas a su quehacer docente.
Estándar 10:	Aprende en forma continua y reflexiona sobre su práctica y su inserción en el sistema educacional.

Estándar 4: *Sabe cómo diseñar e implementar estrategias de enseñanza-aprendizaje adecuadas para los objetivos de aprendizaje y de acuerdo al contexto.*

El futuro profesor o profesora es capaz de planificar la enseñanza teniendo como foco el logro de objetivos de aprendizaje relevantes para los estudiantes y coherentes con el currículo nacional. Considera en su planificación las necesidades, intereses, conocimientos previos, habilidades, competencias tecnológicas y experiencias de los estudiantes y el contexto en que se desarrollará la docencia, incluyendo los resultados de evaluaciones previas. Es capaz de planificar experiencias de aprendizaje y secuencias de actividades, dando a los estudiantes el tiempo, el espacio y los recursos necesarios para aprender. Conoce las estrategias didácticas propias de cada área curricular y disciplina y es capaz de transformar este conocimiento en enseñanza. Incorpora recursos TIC en los diseños, en la implementación curricular y en la evaluación educativa, seleccionando los que son apropiados para favorecer los procesos de enseñanza y aprendizaje. Incorpora en la reflexión sobre su propia práctica la evaluación sistemática de la efectividad de las planificaciones en función del aprendizaje logrado y puede realizar los ajustes necesarios basados en decisiones pedagógicas fundamentadas.

Lo que se manifiesta cuando:

1. Diseña, de manera individual o colectiva, planificaciones de distinto alcance temporal para lograr los aprendizajes esperados de acuerdo al currículo en las distintas áreas.
2. Elabora planificaciones donde las estrategias de enseñanza, las actividades, los recursos y la evaluación son efectivos y coherentes con el logro de los objetivos de aprendizaje.
3. Incorpora en las planificaciones objetivos de aprendizaje y acciones específicas para el inicio, desarrollo y cierre de una clase optimizando el uso del tiempo disponible.
4. Fundamenta las decisiones pedagógicas tomadas en una planificación y evalúa críticamente posibles alternativas para ajustarla o mejorarla de acuerdo a las necesidades de aprendizaje manifestando apertura para recibir u ofrecer retroalimentación.
5. Conoce un repertorio de estrategias metodológicas para enseñar un objetivo.
6. Argumenta sobre la relación positiva entre el diseño sistemático de las actividades pedagógicas, la efectividad de la enseñanza y el logro de aprendizajes e identifica riesgos asociados a realizar planificaciones que no se ajusten al contexto y a los resultados de aprendizaje.
7. Ajusta y modifica planificaciones considerando las características de sus estudiantes, adaptándolas a las necesidades emergentes, a las evaluaciones del proceso y a los resultados de aprendizajes alcanzados.
8. Prepara situaciones de aprendizaje que permitan integrar los objetivos fundamentales transversales cuando es pertinente y establecer conexiones entre los aprendizajes mínimos y transversales a desarrollar en diferentes sectores, así como entre las distintas áreas que conforman un sector disciplinar del currículo.
9. Selecciona TIC que potencian el desarrollo de la enseñanza en cada área curricular fundamentándose en criterios como su aporte al aprendizaje y al desarrollo de habilidades de orden superior (cognitivas, de comunicación, expresión y creación).
10. Utiliza las TIC para apoyar las labores relacionadas con la administración y gestión de su práctica profesional en el establecimiento y en el aula.

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ESTÁNDARES DE MATEMÁTICA

El futuro profesor o profesora:

SISTEMAS NUMÉRICOS Y ÁLGEBRA

Estándar 1: Es capaz de conducir el aprendizaje de los sistemas numéricos N, Z, Q, R y C.

Estándar 2: Es capaz de conducir el aprendizaje de las operaciones del álgebra elemental y sus aplicaciones a la resolución de ecuaciones e inecuaciones.

Estándar 3: Es capaz de conducir el aprendizaje del concepto de función, sus propiedades y representaciones.

Estándar 4: Demuestra competencia disciplinaria en álgebra lineal y es capaz de conducir el aprendizaje de sus aplicaciones en la Matemática escolar.

CÁLCULO

Estándar 5: Es capaz de conducir el aprendizaje de los números reales, sucesiones, sumatorias y series.

Estándar 6: Demuestra competencia disciplinaria en cálculo diferencial y aplicaciones.

Estándar 7: Demuestra competencia disciplinaria en cálculo integral y aplicaciones.

ESTRUCTURAS ALGEBRAICAS

Estándar 8: Es capaz de conducir el aprendizaje de la divisibilidad de números enteros y de polinomios y demuestra competencia disciplinaria en su generalización a la estructura de anillo.

Estándar 9: Demuestra competencia disciplinaria en teoría de grupos y cuerpos.

Estándar 10: Demuestra competencia disciplinaria en conceptos y construcciones fundamentales de la Matemática.

GEOMETRÍA

Estándar 11: Es capaz de conducir el aprendizaje de los conceptos elementales de la Geometría.

Estándar 12: Es capaz de conducir el aprendizaje de transformaciones isométricas y homotecias de figuras en el plano.

Estándar 13: Es capaz de conducir el aprendizaje de los estudiantes en temas referidos a medida de atributos de objetos geométricos y el uso de la trigonometría.

Estándar 14: Es capaz de conducir el aprendizaje de la Geometría analítica plana.

Estándar 15: Es capaz de conducir el aprendizaje de la Geometría del espacio usando vectores y coordenadas.

Estándar 16: Comprende aspectos fundantes de la Geometría euclidiana y algunos modelos básicos de geometrías no euclidianas.

DATOS Y AZAR

Estándar 17: Es capaz de motivar la recolección y estudio de datos y de conducir el aprendizaje de las herramientas básicas de su representación y análisis.

Estándar 18: Es capaz de conducir el aprendizaje de las probabilidades discretas.

Estándar 19: Está preparado para conducir el aprendizaje de las variables aleatorias discretas.

Estándar 20: Está preparado para conducir el aprendizaje de la distribución normal y teoremas límite.

Estándar 21: Está preparado para conducir el aprendizaje de inferencia estadística.

Estándar 2: *Es capaz de conducir el aprendizaje de las operaciones del álgebra elemental y sus aplicaciones a la resolución de ecuaciones e inecuaciones.*

El futuro profesor o profesora está capacitado para conducir el aprendizaje de sus alumnas y alumnos en la comprensión y utilización de expresiones algebraicas y la solución de ecuaciones e inecuaciones que involucran polinomios, logaritmos, potencias, raíces y exponenciales, promoviendo en ellos el desarrollo de habilidades de cálculo, análisis y resolución de problemas. Para esto planifica actividades, analiza recursos pedagógicos y diseña evaluaciones, tomando en cuenta la diversidad en el aula y promoviendo el desarrollo de las capacidades matemáticas de los alumnos y alumnas. Conecta las expresiones algebraicas, ecuaciones e inecuaciones con otros temas del currículo. Analiza y reflexiona acerca de creencias en la enseñanza del álgebra así como de los errores frecuentes que presentan los alumnos y alumnas en el uso de estos contenidos.

Lo que se manifiesta cuando:

1. Construye geométricamente raíces de polinomios.
2. Resuelve problemas que involucran ecuaciones cuadráticas.
3. Resuelve inecuaciones que involucran funciones racionales y valor absoluto.
4. Resuelve ecuaciones e inecuaciones que involucran logaritmos y exponenciales.
5. Conoce errores frecuentes y dificultades en el aprendizaje del valor absoluto, raíces cuadradas y ecuaciones con radicales y se anticipa a ellos en su planificación de actividades.
6. Conoce errores frecuentes en el uso de expresiones polinomiales y racionales y propone actividades para anticiparse a ellos.
7. Reconoce la progresión de los contenidos de expresiones algebraicas en el eje de álgebra y su relación con otros ejes del currículo de Matemática.
8. Relaciona contenidos de expresiones algebraicas con contenidos de otros sectores del currículo.
9. Planifica actividades que permitan hacer surgir la necesidad del uso de ecuaciones lineales con una incógnita, comprender y aplicar los procedimientos involucrados en su resolución, así como analizar el conjunto solución.
10. Planifica actividades relativas a ecuaciones e inecuaciones, incorporando el uso de programas computacionales.
11. Analiza textos escolares, guías, y otros recursos pedagógicos para la planificación de clases y actividades relacionadas con expresiones radicales.
12. Es capaz de gestionar clases para introducir los temas de raíces y logaritmos.
13. Es capaz de gestionar la clase para que sus estudiantes describan la solución de inecuaciones utilizando lenguaje de conjuntos y conectivos lógicos.
14. Reflexiona respecto a las habilidades que desarrollan los estudiantes al realizar actividades de manejo de expresiones algebraicas.
15. Diseña actividades que permitan evaluar contenidos de expresiones algebraicas.
16. Reflexiona sobre creencias y actitudes acerca de las expresiones algebraicas y sus consecuencias en la práctica docente.
17. Elabora instrumentos para evaluar contenidos de álgebra a alumnos con diferentes ritmos de aprendizaje.
18. Reflexiona acerca de estrategias de gestión de clase para desarrollar las capacidades matemáticas de todos sus alumnos y alumnas.

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Australia (2016)

Asociación Australiana de docentes de Matemáticas

“These Standards were originally adopted by AAMT Council in 2002 as representing a consensus view, by the profession for the profession, describing the knowledge, skills and attributes required for good teaching of mathematics “

THE AUSTRALIAN ASSOCIATION OF MATHEMATICS TEACHERS

Standards for Excellence

in Teaching Mathematics
in Australian Schools

2006 edition

DOMAIN 1: PROFESSIONAL KNOWLEDGE

Excellent teachers of mathematics have a strong knowledge base to draw on in all aspects of their professional work, including their decision making, planning and interactions. Their knowledge base includes knowledge of students, how mathematics is learned, what affects students’ opportunities to learn mathematics and how the learning of mathematics can be enhanc-ed. It also includes sound knowledge and appreciation of mathematics appropriate to the grade level and/or mathematics subjects they teach.

1.1 Knowledge… of students

Excellent teachers of mathematics have a thorough knowledge of the students they teach. This includes knowledge of students’ social and cultural contexts, the mathematics they know and use, their preferred ways of learning, and how confident they feel about learning mathematics.

1.2 Knowledge… of mathematics

Excellent teachers of mathematics have a sound, coherent knowledge of the mathematics appropriate to the student level they teach, and which is situated in their knowledge and understanding of the broader mathematics curriculum. They understand how mathematics is represented and communicated, and why mathematics is taught. They are confident and competent users of mathematics who understand connections within mathematics, between mathematics and other subject areas, and how mathematics is related to society.

1.3 Knowledge… of students’ learning of mathematics

Excellent teachers of mathematics have rich knowledge of how students learn mathematics. They have an understanding of current theories relevant to the learning of mathematics. They have knowledge of the mathematical development of students including learning sequences, appropriate representations, models and language. They are aware of a range of effective strategies and techniques for: teaching and learning mathematics; promoting enjoyment of learning and positive attitudes to mathematics; utilising information and communication technologies; encouraging and enabling parental involvement; and for being an effective role model for students and the community in the ways they deal with mathematics.

5 páginas en total

"As standards for excellence, the AAMT Standards provide targets to which all teachers of mathematics can aspire and work towards in their professional development. For those teachers who wish to be acknowledged as reaching the high standards described by the Standards, the AAMT has designed, tested and established the program of assessment that allows them to be awarded the AAMT's Highly Accomplished Teacher of Mathematics credential. "

DOMAIN 2: PROFESSIONAL ATTRIBUTES

Excellent teachers of mathematics are committed and enthusiastic professionals who continue to extend their knowledge of both mathematics and student learning. They work creatively and constructively within a range of 'communities' inside and beyond the school and set high, achievable goals for themselves and their students. These teachers exhibit personal approaches characterised by caring and respect for others.

2.1 Personal attributes

The work of excellent teachers of mathematics reflects a range of personal attributes that assists them to engage students in their learning. Their enthusiasm for mathematics and its learning characterises their work. These teachers have a conviction that all students can learn mathematics. They are committed to maximising students' opportunities to learn mathematics and set high achievable standards for the learning of each student. They aim for students to become autonomous and self directed learners who enjoy mathematics. These teachers exhibit care and respect for their students.

2.2 Personal professional development

Excellent teachers of mathematics are committed to the continual improvement of their teaching practice and take opportunities for personal professional development. They undertake sustained, purposeful professional growth in their own knowledge, understanding and skills in mathematics, and in the teaching and learning of mathematics. The professional development they undertake enables them to develop informed views about relevant current trends (including teaching and learning resources, technologies, and changes to the curriculum with which they work) and to further their teaching expertise. They are involved in professional development processes that include collegial interaction, professional reading and active exploration of new teaching ideas, practices and resources in the classroom. They reflect on practice and the new knowledge they gain, and learn from their experiences.

2.3 Community responsibilities

Excellent teachers of mathematics are active contributors to the range of communities relevant to their professional work. They are positive advocates for mathematics and its learning in the school and the wider community. They ensure effective interaction with families including provision of information about students' learning and progress. They offer strategies for assisting students' mathematical development outside the classroom. They create and take opportunities to involve students in mathematical activities beyond the classroom in contexts of interest and relevance to the students. They contribute to the improvement of mathematics teaching by actively engaging and collaborating with colleagues both individually and in teams – learning; sharing insights, practices and resources; supporting and mentoring others; and providing feedback. They actively participate in school decision-making.

DOMAIN 3: PROFESSIONAL PRACTICE

Excellent teachers of mathematics are purposeful in making a positive difference to the learning outcomes, both cognitive and affective, of the students they teach. They are sensitive and responsive to all aspects of the context in which they teach. This is reflected in the learning environments they establish, the lessons they plan, their uses of technologies and other resources, their teaching practices, and the ways in which they assess and report on student learning.

3.1 The learning environment

Excellent teachers of mathematics establish an environment that maximises students' learning opportunities. The psychological, emotional and physical needs of students are addressed and the teacher is aware of, and responds to, the diversity of students' individual needs and talents. Students are empowered to become independent learners. They are motivated to improve their understanding of mathematics and develop enthusiasm for, enjoyment of, and interest in mathematics. In an inclusive and caring atmosphere of trust and belonging, active engagement with mathematics is valued, communication skills fostered, and co-operative and collaborative efforts encouraged.

3.2 Planning for learning

Excellent teachers of mathematics plan for coherently organised learning experiences that have the flexibility to allow for spontaneous, self-directed learning. These learning experiences involve substantive mathematics. They enable students to develop new mathematical understandings that build on and enrich their knowledge and appreciation of mathematics. A variety of appropriate teaching strategies is incorporated in the intended learning experiences, enhanced by available technologies and other resources. Students' backgrounds and prior mathematical knowledge are taken into account. Students are provided with opportunities to explore and apply mathematics across key learning areas and beyond the school setting.

3.3 Teaching in action

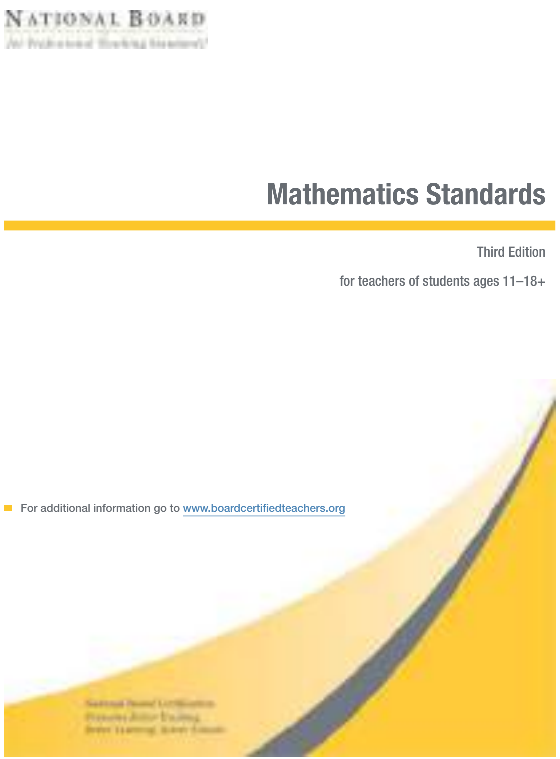
Excellent teachers of mathematics arouse curiosity, challenge students' thinking, and engage them actively in learning. They initiate purposeful mathematical dialogue with and among students. As facilitators of learning, excellent teachers negotiate mathematical meaning and model mathematical thinking and reasoning. Their teaching promotes, expects and supports creative thinking, mathematical risk-taking in finding and explaining solutions, and involves strategic intervention and provision of appropriate assistance.

3.4 Assessment

Excellent teachers of mathematics regularly assess and report student learning outcomes, both cognitive and affective, with respect to skills, content, processes, and attitudes. They use a range of assessment strategies that are fair, inclusive and appropriate to both the students and the learning context. They maintain on-going, informative records of student learning outcomes that are used to map student progress and to plan appropriate future learning experiences. The excellent teacher of mathematics provides constructive, purposeful and timely feedback to students and their parents, and to school authorities, as required.

EEUU - National Board (2016)

Mathematics Standards for teachers of students ages 11–18+



The National Board for Professional Teaching Standards (National Board) is a not-for-profit professional organization, created and governed by practicing teachers and their advocates. The founding mission of the National Board is to advance the quality of teaching and learning by

- maintaining high and rigorous standards for what accomplished teachers should know and be able to do;
- providing a national voluntary system certifying teachers who meet these standards; and
- advocating related education reforms to integrate National Board Certification into American education and to capitalize on the expertise of National Board Certified Teachers.

EEUU - National Board (2016)

Mathematics Standards for teachers of students ages 11–18+

Five Core Propositions (generales): Teachers….

1. are committed to students and their learning.
2. know the subjects they teach and how to teach those subjects to students.
3. are responsible for managing and monitoring student learning.
4. think systematically about their practice and learn from experience.
5. are members of learning communities.

Recognized as the “gold standard” in teacher certification, the National Board believes higher standards for teachers means better learning for students.

The National Board believes that board certification should become the norm, not the exception, and should be fully integrated into the fabric of the teaching profession. In other professions, such as medicine, engineering, and architecture, board certification has helped to create a culture of accomplished practice and is a major reason why those professions are held in such high regard by the public. Those professions did what teaching must now do: strengthen the coherent pipeline of preparation that begins in pre- service and continues through board certification and beyond, with each step engineered to help teachers develop toward accomplished.

EEUU (2016)

Mathematics Standards for teachers of students ages 11–18+

The National Board for Professional Teaching Standards

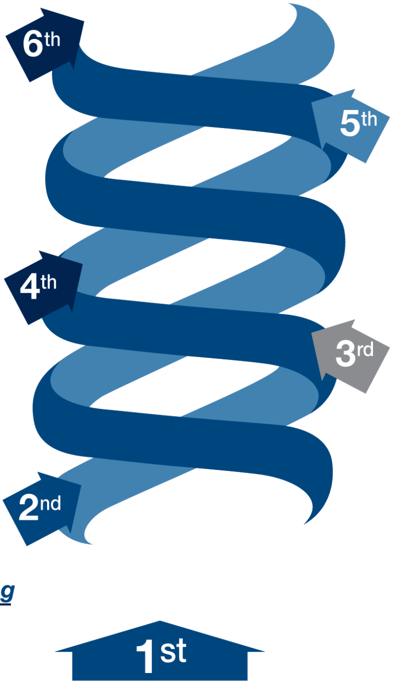
The Architecture of Accomplished Teaching:

What is underneath the surface?

Set new high and worthwhile goals that are appropriate for these students at this time

Evaluate student learning in light of the goals and the instruction

Set high, worthwhile goals appropriate for these students, at this time, in this setting



Reflect on student learning, the effectiveness of the instructional design, particular concerns, and issues

Implement instruction designed to attain those goals

***Your Students** - Who are they? Where are they now? What do they need and in what order do they need it? Where should I begin?*

Five Core Propositions

- **Teachers are committed to students and their learning**
- **Teachers know the subjects they teach and how to teach those subjects to students**
- **Teachers are responsible for managing and monitoring student learning**
- **Teachers think systematically about their practice and learn from experience**
- **Teachers are members of learning communities**

EEUU (2016)

Mathematics Standards for teachers of students ages 11–18+

The National Board for Professional Teaching Standards

Standard I: Commitment to Mathematics Learning of All Students

Accomplished mathematics teachers acknowledge and value the individuality and worth of each student, believe that every student can learn and use mathematics, and are dedicated to their success. Accomplished mathematics teachers are committed to the fair and equitable treatment of all students—especially in their learning of mathematics.

Knowledge of Mathematics, Students, and Teaching

Standard II: Knowledge of Mathematics

Accomplished mathematics teachers have a deep and broad knowledge of the concepts, principles, techniques, and reasoning methods of mathematics, and they use this knowledge to inform curricular goals and shape their instruction and assessment. They understand significant connections among mathematical ideas and the applications of these ideas to problem solving in mathematics, in other disciplines, and in the world outside of school.

Standard III: Knowledge of Students

Accomplished teachers use their knowledge of human development and individual students to guide their planning and instructional decisions. They understand the impact of prior mathematical knowledge, home life, cultural background, individual learning differences, student attitudes and aspirations, and community expectations and values on students and their mathematics learning.

Standard IV: Knowledge of the Practice of Teaching

Accomplished mathematics teachers use their knowledge of pedagogy along with their knowledge of mathematics and student learning to inform curricular decisions; select, design, and develop instructional strategies and assessment plans; and choose materials and resources for mathematics instruction. Accomplished mathematics teachers stimulate and facilitate student learning by using a wide range of practices.

EEUU (2016)

Mathematics Standards for teachers of students ages 11–18+

The National Board for Professional Teaching Standards

The Teaching of Mathematics

Standard V: Learning Environment

Accomplished mathematics teachers create environments in which students are active learners, show willingness to take intellectual risks, develop self-confidence, and value mathematics. This environment fosters student learning of mathematics.

Standard VI: Ways of Thinking Mathematically

Accomplished mathematics teachers develop their own and their students' abilities to reason and think mathematically—to investigate and explore patterns, to discover structures and establish mathematical relationships, to formulate and solve problems, to justify and communicate conclusions, and to question and extend those conclusions.

Standard VII: Assessment

Accomplished mathematics teachers integrate a range of assessment methods into their instruction to promote the learning of all students by designing, selecting, and ethically employing assessments that align with educational goals. They provide opportunities for students to reflect on their strengths and weaknesses in order to revise, support, and extend their individual performance.

Professional Development and Outreach

Standard VIII: Reflection and Growth

To improve practice, accomplished mathematics teachers regularly reflect on what they teach, how they teach, and how their teaching impacts student learning. They keep abreast of changes and learn new mathematics and mathematical pedagogy, continually improving their knowledge and practice.

Standard IX: Families and Communities

Accomplished mathematics teachers collaborate with families and communities to support student engagement in learning mathematics. They help various communities, within and outside the school building, understand the role of mathematics and mathematics instruction in today's world.

Standard X: Professional Community

Accomplished mathematics teachers continually collaborate with other teachers and education professionals to strengthen the school's mathematics program, promote program quality and continuity across grade levels and courses, and improve knowledge and practice in the field of mathematics education.

EEUU (2016)

Mathematics Standards for teachers of students ages 11–18+

The National Board for Professional Teaching Standards

Standard V
Learning Environment

Accomplished mathematics teachers create environments in which students are active learners, show willingness to take intellectual risks, develop self-confidence, and value mathematics. This environment fosters student learning of mathematics.

Accomplished teachers use their knowledge of how students learn to create a stimulating and productive environment in which students are empowered to do mathematics. Teachers foster a respectful, engaging, and cooperative atmosphere for learning. They help students learn about learning mathematics. From the beginning of the school year, teachers engage their students in creating a community of learners in which students value taking intellectual risks.

Standard VI
Ways of Thinking Mathematically

Accomplished mathematics teachers develop their own and their students' abilities to reason and think mathematically—to investigate and explore patterns, to discover structures and establish mathematical relationships, to formulate and solve problems, to justify and communicate conclusions, and to question and extend those conclusions.

Accomplished teachers bring insight about mathematics to students, including new perspectives on standard problems and unexpected connections among different fields. Teachers are proficient not only in solving problems, but also in making students aware of different strategies for solving a problem, as well as the relative merits of each. They have the confidence to help students face uncertainties and make strategic decisions in exploring unknown territories.

Accomplished teachers know that mathematics is a discipline of concepts, principles, procedures, and reasoning processes. Thinking mathematically includes representing, modeling, proving, experimenting, conjecturing, classifying, visualizing, and computing. In the classrooms of accomplished teachers, students are engaged in identifying patterns; solving problems; reasoning; forming and testing conjectures, justification and proof; and communicating results. Students search for connections and solve problems, while reflecting on both the mathematics and their own thought processes.

Accomplished teachers recognize that important general concepts and reasoning methods undergird the development of mathematical power. They model mathematical reasoning as they work with students and encourage students to question processes and challenge the validity of particular approaches. Students make conjectures and justify or refute them, formulate convincing arguments, and draw logical conclusions. Sound reasoning—not an edict from the teacher—is the arbiter of mathematical correctness. In short, students become mathematically empowered as they learn to think, reason, and communicate mathematically.

EEUU (2012)

Standards for Mathematics Teacher Preparation

The National Council of Teachers of Mathematics

Standard 1: Content Knowledge

Effective teachers of secondary mathematics demonstrate and apply knowledge of major mathematics concepts, algorithms, procedures, connections, and applications within and among mathematical content domains.

Preservice teacher candidates:

1a) Demonstrate and apply knowledge of major mathematics concepts, algorithms, procedures, applications in varied contexts, and connections within and among mathematical domains (Number, Algebra, Geometry, Trigonometry, Statistics, Probability, Calculus, and Discrete Mathematics) as outlined in the *NCTM CAEP Mathematics Content for Secondary*.

Standard 2: Mathematical Practices

Effective teachers of secondary mathematics solve problems, represent mathematical ideas, reason, prove, use mathematical models, attend to precision, identify elements of structure, generalize, engage in mathematical communication, and make connections as essential mathematical practices. They understand that these practices intersect with mathematical content and that understanding relies on the ability to demonstrate these practices within and among mathematical domains and in their teaching.

Preservice teacher candidates:

2a) Use problem solving to develop conceptual understanding, make sense of a wide variety of problems and persevere in solving them, apply and adapt a variety of strategies in solving problems confronted within the field of mathematics and other contexts, and formulate and test conjectures in order to frame generalizations.

2b) Reason abstractly, reflectively, and quantitatively with attention to units, constructing viable arguments and proofs, and critiquing the reasoning of others; represent and model generalizations using mathematics; recognize structure and express regularity in patterns of mathematical reasoning; use multiple representations to model and describe mathematics; and utilize appropriate mathematical vocabulary and symbols to communicate

EEUU (2012)

Standards for Mathematics Teacher Preparation

The National Council of Teachers of Mathematics

Standard 4: Mathematical Learning Environment

Effective teachers of secondary mathematics exhibit knowledge of adolescent learning, development, and behavior. They use this knowledge to plan and create sequential learning opportunities grounded in mathematics education research where students are actively engaged in the mathematics they are learning and building from prior knowledge and skills. They demonstrate a positive disposition toward mathematical practices and learning, include culturally relevant perspectives in teaching, and demonstrate equitable and ethical treatment of and high expectations for all students. They use instructional tools such as manipulatives, digital tools, and virtual resources to enhance learning while recognizing the possible limitations of such tools.

Preservice teacher candidates:

4a) Exhibit knowledge of adolescent learning, development, and behavior and demonstrate a positive disposition toward mathematical processes and learning.

4b) Plan and create developmentally appropriate, sequential, and challenging learning opportunities grounded in mathematics education research in which students are actively engaged in building new knowledge from prior knowledge and experiences.

4c) Incorporate knowledge of individual differences and the cultural and language diversity that exists within classrooms and include culturally relevant perspectives as a means to motivate and engage students.

4d) Demonstrate equitable and ethical treatment of and high expectations for all students.

EEUU (2012)

Standards for Mathematics Teacher Preparation

The National Council of Teachers of Mathematics

Contenidos asociados

B. Middle Grades Mathematics Teachers

All middle grades mathematics teachers should be prepared with depth and breadth in the following mathematical domains: Number, Algebra, Geometry, Trigonometry, Statistics, Probability, and Calculus. All teachers certified in middle grades mathematics should know, understand, teach, and be able to communicate their mathematical knowledge with the breadth of understanding reflecting the following competencies for each of these domains.

B.1. Number Systems

To be prepared to develop student mathematical proficiency, all middle grades mathematics teachers should know the following topics related to number systems with their content understanding and mathematical practices supported by appropriate technology and varied representational tools, including concrete models:

B.1.1 Structure, properties, relationships, operations, and representations, including standard and non-standard algorithms, of numbers and number systems including whole, integer, rational, irrational, real, and complex numbers

B.1.2 Fundamental ideas of number theory (divisors, factors and factorization, primes, composite numbers, greatest common factor, and least common multiple)

B.1.3 Quantitative reasoning and relationships that include ratio, rate, and proportion and the use of units in problem situations

B.1.4 Vector and matrix operations, modeling, and applications

B.1.5 Historical development and perspectives of number, number systems, and quantity including contributions of significant figures and diverse cultures

México (2008)

ACUERDO número 447 por el que se establecen las competencias docentes para quienes impartan educación media superior en la modalidad escolarizada

Artículo 3.- Las competencias docentes son las que formulan las cualidades individuales, de carácter ético, académico, profesional y social que debe reunir el docente de la EMS, y consecuentemente definen su perfil.
Artículo 4.- Las competencias y sus principales atributos que han de definir el Perfil del Docente del SNB, son las que se establecen a continuación:

1. Organiza su formación continua a lo largo de su trayectoria profesional.

- Reflexiona e investiga sobre la enseñanza y sus propios procesos de construcción del conocimiento.
- Incorpora nuevos conocimientos y experiencias al acervo con el que cuenta y los traduce en estrategias de enseñanza y de aprendizaje.
- Se evalúa para mejorar su proceso de construcción del conocimiento y adquisición de competencias, y cuenta con una disposición favorable para la evaluación docente y de pares.
- Aprende de las experiencias de otros docentes y participa en la conformación y mejoramiento de su comunidad académica.
- Se mantiene actualizado en el uso de la tecnología de la información y la comunicación.
- Se actualiza en el uso de una segunda lengua.

2. Domina y estructura los saberes para facilitar experiencias de aprendizaje significativo.

- Argumenta la naturaleza, los métodos y la consistencia lógica de los saberes que imparte.
- Explicita la relación de distintos saberes disciplinares con su práctica docente y los procesos de aprendizaje de los estudiantes.
- Valora y explicita los vínculos entre los conocimientos previamente adquiridos por los estudiantes, los que se desarrollan en su curso y aquellos otros que conforman un plan de estudios.

3. Planifica los procesos de enseñanza y de aprendizaje atendiendo al enfoque por competencias, y los ubica en contextos disciplinares, curriculares y sociales amplios.

- Identifica los conocimientos previos y necesidades de formación de los estudiantes, y desarrolla estrategias para avanzar a partir de ellas.
- Diseña planes de trabajo basados en proyectos e investigaciones disciplinarios e interdisciplinarios orientados al desarrollo de competencias.
- Diseña y utiliza en el salón de clases materiales apropiados para el desarrollo de competencias.
- Contextualiza los contenidos de un plan de estudios en la vida cotidiana de los estudiantes y la realidad social de la comunidad a la que pertenecen.

México (2008)

ACUERDO número 447 por el que se establecen las competencias docentes para quienes impartan educación media superior en la modalidad escolarizada

Artículo 3.- Las competencias docentes son las que formulan las cualidades individuales, de carácter ético, académico, profesional y social que debe reunir el docente de la EMS, y consecuentemente definen su perfil.
Artículo 4.- Las competencias y sus principales atributos que han de definir el Perfil del Docente del SNB, son las que se establecen a continuación:

4. Lleva a la práctica procesos de enseñanza y de aprendizaje de manera efectiva, creativa e innovadora a su contexto institucional.

- Comunica ideas y conceptos con claridad en los diferentes ambientes de aprendizaje y ofrece ejemplos pertinentes a la vida de los estudiantes.
- Aplica estrategias de aprendizaje y soluciones creativas ante contingencias, teniendo en cuenta las características de su contexto institucional, y utilizando los recursos y materiales disponibles de manera adecuada.
- Promueve el desarrollo de los estudiantes mediante el aprendizaje, en el marco de sus aspiraciones, necesidades y posibilidades como individuos, y en relación a sus circunstancias socioculturales.
- Provee de bibliografía relevante y orienta a los estudiantes en la consulta de fuentes para la investigación.
- Utiliza la tecnología de la información y la comunicación con una aplicación didáctica y estraté gica en distintos ambientes de aprendizaje.

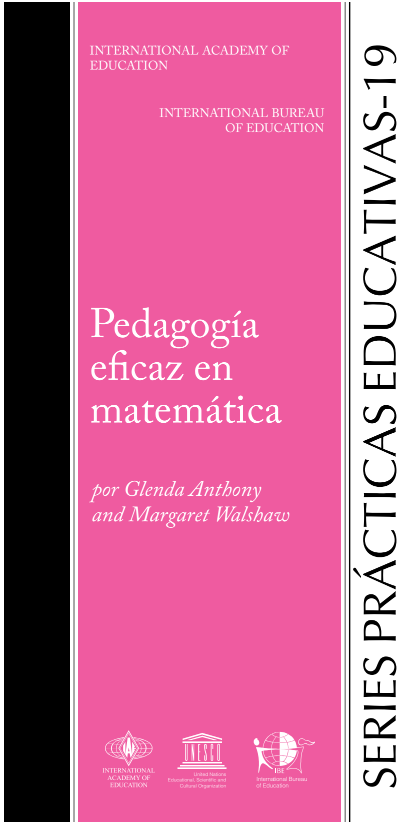
5. Evalua los procesos de enseñanza y de aprendizaje con un enfoque formativo.

- Establece criterios y métodos de evaluación del aprendizaje con base en el enfoque de competencias, y los comunica de manera clara a los estudiantes.
- Da seguimiento al proceso de aprendizaje y al desarrollo académico de los estudiantes.
- Comunica sus observaciones a los estudiantes de manera constructiva y consistente, y sugiere alternativas para su superación.
- Fomenta la autoevaluación y coevaluación entre pares académicos y entre los estudiantes para afianzar los procesos de enseñanza y de aprendizaje.

- …

UNESCO (2000)

Academia internacional de Educación



7. Comunicación matemática

Los docentes efectivos son capaces de facilitar el diálogo enfocado a la argumentación matemática en clase.

Resultados de la investigación

Los docentes eficaces alientan a sus estudiantes a explicar y justificar sus soluciones. Piden que tomen una posición y la defiendan ante demandas matemáticas contrarias, expuestas por otros estudiantes. Ellos supervisan los intentos de los estudiantes para examinar conjeturas, desacuerdos y contraargumentos. Con su orientación, los estudiantes aprenden cómo utilizar ideas matemáticas, lenguaje y métodos. Cuando la atención cambia de normas de procedimiento para dar sentido a la matemática, los estudiantes se preocupan menos de hallar respuestas y más de pensar en qué los conduce a esas respuestas.

Intentos de supervisión de modos matemáticos del habla y pensamiento

Los estudiantes deben aprender a comunicarse matemáticamente, dar explicaciones matemáticas concretas y justificar sus soluciones. Los docentes eficientes animan a sus estudiantes a comunicar sus ideas de forma oral, escrita y utilizando una variedad de representaciones.

Reafirmar es un modo de guiar a los estudiantes en el uso de convenciones matemáticas. La reafirmación implica repetir, reformular o expandir el habla del estudiante. Los docentes pueden utilizarla para:

1. Resaltar ideas que vienen directamente de los estudiantes.
2. Ayudar a desarrollar la comprensión de los estudiantes que está implícita en esas ideas.
3. Agregar nuevas ideas o llevar la discusión en otra dirección.

Desarrollar habilidades de argumentación matemática

Para guiar a los estudiantes en las formas de argumentación matemática, los docentes eficientes los animan a tomar y defender posiciones en contra de ideas alternativas; sus estudiantes se acostumbran a escuchar las ideas de otros y utilizan el debate para resolver conflictos y llegar a acuerdos comunes.

En el siguiente episodio, una clase estuvo discutiendo la idea de que las fracciones pueden ser convertidas en decimales. Bruno y Gina han estado desarrollando habilidades de argumentación matemática durante esta discusión. La maestra entonces se dirige a la clase:

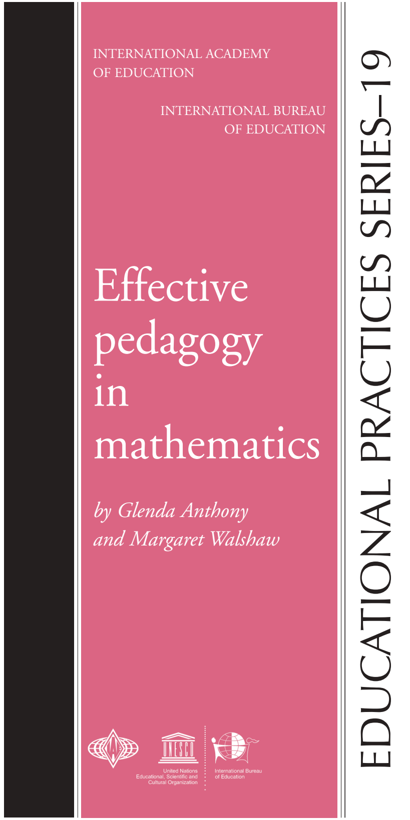
> "Bien, ahora espero que estén escuchando porque lo que dijeron Gina y Bruno fue muy importante. Bruno hizo una conjetura y Gina la probó por él, y en base a sus pruebas él revisa sus conjeturas porque para eso son las conjeturas. Eso significa que uno piensa que ve un patrón, de modo que llegará a una declaración que cree es cierta, pero aún no está convencido. Basado en una evidencia adicional, Bruno revisó su conjetura y luego podría revisar nuevamente lo que declaró en un principio o algo totalmente nuevo. Pero están haciendo algo importante. Están buscando patrones y tratando de llegar a generalizaciones".
>
> *O'Connor (2001, pp. 155–156)*

Esta maestra sostuvo el flujo de ideas de sus estudiantes y supo cuándo intervenir o no en la discusión, cuándo presionar su comprensión, cuándo resolver la competencia de reclamos de los estudiantes y cuándo abordar malentendidos o confusiones. Mientras los estudiantes aprendían sobre la argumentación matemática y descubrían qué hace que un argumento sea convincente, ella escuchaba atentamente a las ideas e información de los estudiantes. Es importante destacar que ella se abstuvo de emitir sus propias explicaciones hasta que fueron necesarias.

Lectura sugerida: Lobato, Clarke, & Ellis, 2005; O'Connor, 2001; Yackel, Cobb, & Wood, 1998.

UNESCO (2000)

International Academy Of Education



7. Mathematical Communication

Effective teachers are able to facilitate classroom dialogue that is focused on mathematical argumentation.

Research findings

Effective teachers encourage their students to explain and justify their solutions. They ask them to take and defend positions against the contrary mathematical claims of other students. They scaffold student attempts to examine conjectures, disagreements, and counterarguments. With their guidance, students learn how to use mathematical ideas, language, and methods. As attention shifts from procedural rules to making sense of mathematics, students become less preoccupied with finding the answers and more with the thinking that leads to the answers.

Scaffolding attempts at mathematical ways of speaking and thinking

Students need to be taught how to communicate mathematically, give sound mathematical explanations, and justify their solutions. Effective teachers encourage their students to communicate their ideas orally, in writing, and by using a variety of representations.

Revoicing is one way of guiding students in the use of mathematical conventions. Revoicing involves repeating, rephrasing, or expanding on student talk. Teachers can use it (i) to highlight ideas that have come directly from students, (ii) to help develop students' understandings that are implicit in those ideas, (iii) to negotiate meaning with their students, and (iv) to add new ideas, or move discussion in another direction.

Developing skills of mathematical argumentation

To guide students in the ways of mathematical argumentation, effective teachers encourage them to take and defend positions against alternative views; their students become accustomed to listening to the ideas of others and using debate to resolve conflict and arrive at

developing the skills of mathematical argumentation during this discussion. The teacher then speaks to the class:

> Teacher: Great, now I hope you're listening because what Gina and Bruno said was very important. Bruno made a conjecture and Gina tested it for him. And based on her tests he revised his conjecture because that's what a conjecture is. It means that you think that you're seeing a pattern so you're gonna come up with a statement that you think is true, but you're not convinced yet. But based on her further evidence, Bruno revised his conjecture. Then he might go back to revise it again, back to what he originally said or to something totally new. But they're doing something important. They're looking for patterns and they're trying to come up with generalizations.
>
> *O'Connor (2001, pp. 155–156)*

This teacher sustained the flow of student ideas, knowing when to step in and out of the discussion, when to press for understanding, when to resolve competing student claims, and when to address misunderstandings or confusion. While the students were learning mathematical argumentation and discovering what makes an argument convincing, she was listening attentively to student ideas and information. Importantly, she withheld her own explanations until they were needed.

Suggested readings: Lobato, Clarke, & Ellis, 2005; O'Connor, 2001; Yackel, Cobb, & Wood, 1998.

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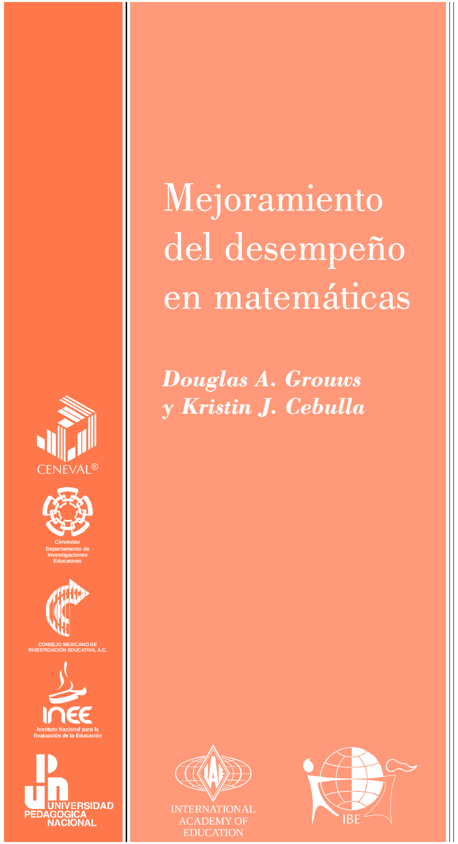

SERIE PRÁCTICAS EDUCATIVAS - 4

2. Enfoque significativo

Enfocar la enseñanza en el desarrollo significativo de los conceptos matemáticos importantes incrementa el nivel de aprendizaje del estudiante.

Resultados de la investigación

La historia de la investigación sobre los efectos de la enseñanza en la comprensión de los conceptos matemáticos es larga. Desde los trabajos de Wiliam Brownell en la década de los cuarenta, las investigaciones han revelado consistentemente que poner énfasis en la enseñanza de los conceptos significativos tiene efectos positivos en el aprendizaje del estudiante, incluyendo un mejor aprovechamiento inicial, mayor retención y un incremento en la probabilidad de que las ideas sean usadas en nuevas situaciones. Estos resultados también se han encontrado en zonas de alta pobreza.

En el aula

Como se podría esperar, el término "enseñanza significativa" ha variado de estudio en estudio y ha evolucionado a través del tiempo. Los maestros querrán conocer cómo sus numerosas interpretaciones pueden ser incorporadas a su práctica en el aula:

- *Poner énfasis en el significado matemático de las ideas*, incluyendo la manera como la idea, concepto o habilidad se conecta en múltiples vías con otras ideas matemáticas, de forma razonable y lógicamente consistente. De este modo, para la resta se resalta la relación inversa o de "deshacer" entre ella y la suma. En general, el acento en el significado era común en las investigaciones tempranas en esta área, a finales de la década de 1930, y su propósito era evitar que las ideas matemáticas más importantes fueran enseñadas con menor atención en comparación con el énfasis puesto en el uso y la utilidad de las matemáticas en la vida diaria.
- *Crear un contexto de aprendizaje en el aula en el cual los estudiantes puedan construir el significado de los conceptos matemáticos*. Los alumnos pueden aprender matemáticas tanto en contextos vinculados directamente con situaciones de la vida real como en aquellos puramente matemáticos. La abstracción del ambiente de aprendizaje y la forma como los estudiantes se relacionan con él deben de ser regulados con cuidado, vigilados de cerca y escogidos concienzudamente, además de tomar en cuenta los intereses y la trayectoria de los estudiantes. Las matemáticas que se enseñan y se aprenden deben parecer razonables; así tendrán sentido para los estudiantes. Un factor decisivo en la enseñanza mediante significados es la conexión de nuevas ideas y habilidades con el conocimiento y las experiencias pasadas.
- *Hacer explícitos los vínculos entre las matemáticas y otras materias*. La instrucción podría relacionar, por ejemplo, las habilidades para la colección y representación de información con encuestas de opinión pública en estudios sociales, o bien se podría vincular el concepto de variación directa en matemáticas con el de fuerza en física, para ayudar a establecer un referente de la idea en el mundo real.
- *Poner atención a los significados y a la comprensión de los estudiantes*. La manera en que se conciben las ideas varía entre los estudiantes, al igual que sus métodos para resolver problemas y dar seguimiento a los procedimientos. Los maestros deben construir sobre las nociones y los métodos intuitivos al diseñar e implementar la enseñanza.

Referencias: Aubrey (1997); Brownell (1945, 1947); Carpenter *et al.* (1998); Cobb *et al.* (1991); Fuson (1992); Good, Grouws y Ebmeier (1983); Hiebert y Carpenter (1992); Hiebert y Wearne (1996); Hiebert *et al.* (1997); Kamii (1985, 1989, 1994); Knapp, Shields y Turnbull (1995); Koehler y Grouws (1992); Skemp (1978); Van Engen (1949), y Wood y Sellers (1996, 1997).

UNESCO (2000)

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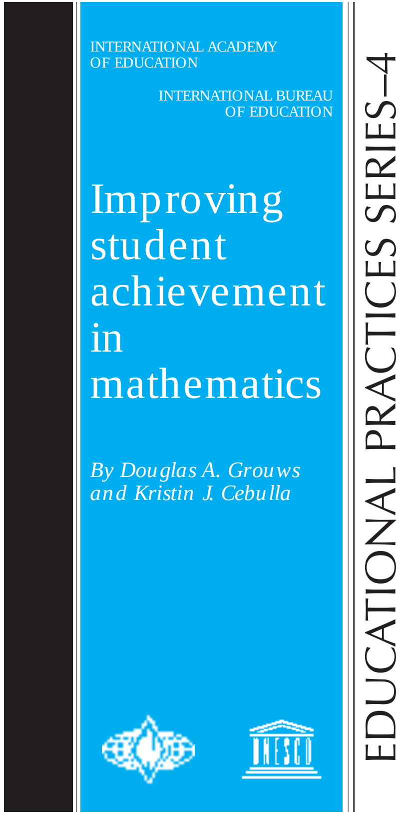

2. Focus on meaning

Focusing instruction on the meaningful development of important mathematical ideas increases the level of student learning.

Research findings

There is a long history of research, going back to the 1940s and the work of William Brownell, on the effects of teaching for meaning and understanding in mathematics. Investigations have consistently shown that an emphasis on teaching for meaning has positive effects on student learning, including better initial learning, greater retention and an increased likelihood that the ideas will be used in new situations. These results have also been found in studies conducted in high-poverty areas.

In the classroom

As might be expected, the concept of 'teaching for meaning' has varied somewhat from study to study, and has evolved over time. Teachers will want to consider how various interpretations of this concept can be incorporated into their classroom practice.

- *Emphasize the mathematical meanings of ideas, including how the idea, concept or skill is connected in multiple ways to other mathematical ideas in a logically consistent and sensible manner.* Thus, for subtraction, emphasize the inverse, or 'undoing', relationship between it and addition. In general, emphasis on meaning was common in early research in this area in the late 1930s, and its purpose was to avoid the mathematical meaningfulness of the ideas taught receiving only minor attention compared to a heavy emphasis on the social uses and utility of mathematics in everyday life.
- *Create a classroom learning context in which students can construct meaning.* Students can learn important mathematics both in contexts that are closely connected to real-life situations and in those that are purely mathematical. The abstractness of a learning environment and how students relate to it must be carefully regulated, closely monitored and thoughtfully chosen. Consideration should be given to students' interests and backgrounds. The mathematics taught and learned must seem reasonable to students and make sense to them. An important factor in teaching for meaning is connecting the new ideas and skills to students' past knowledge and experience.
- *Make explicit the connections between mathematics and other subjects.* For example, instruction could relate data-gathering and data-representation skills to public opinion polling in social studies. Or, it could relate the mathematical concept of direct variation to the concept of force in physics to help establish a real-world referent for the idea.
- *Attend to student meanings and student understanding in instruction.* Students' conceptions of the same idea will vary, as will their methods of solving problems and carrying out procedures. Teachers should build on students' intuitive notions and methods in designing and implementing instruction.

References: Aubrey, 1997; Brownell, 1945, 1947; Carpenter et al., 1998; Cobb et al., 1991; Fuson, 1992; Good, Grouws & Ebmeier, 1983; Hiebert & Carpenter, 1992; Hiebert & Wearne, 1996; Hiebert et al., 1997; Kamii, 1985, 1989, 1994; Knapp, Shields & Turnbull, 1995; Koehler & Grouws, 1992; Skemp, 1978; Van Engen, 1949; Wood & Sellers, 1996, 1997.

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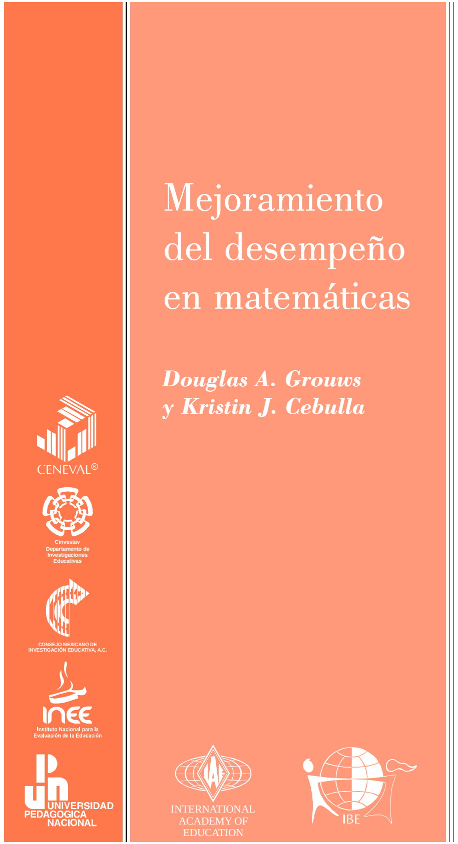


SERIE PRÁCTICAS EDUCATIVAS - 4

5. Apertura a la solución de problemas y a la interacción entre los estudiantes

La enseñanza que aprovecha la intuición de los estudiantes para la solución de problemas puede incrementar el aprendizaje, especialmente cuando se combina con oportunidades para la interacción y la discusión entre ellos.

Resultados de la investigación

Resultados recientes del estudio TIMSS revelan que en las aulas japonesas se usan intensamente durante el tiempo de clase métodos de solución aportados por los estudiantes. Esta misma técnica de enseñanza aparece en muchos proyectos de investigación estadounidenses exitosos. Los estudios revelan claramente dos principios importantes que se relacionan con el desarrollo del entendimiento conceptual profundo de los estudiantes en las matemáticas:

- Primero, el aprovechamiento y entendimiento de los estudiantes mejora significativamente cuando los maestros son concientes de cómo sus alumnos construyen el conocimiento, están familiarizados con los métodos intuitivos de solución que los estudiantes usan cuando resuelven problemas y utilizan este conocimiento para planear y conducir la enseñanza de las matemáticas. Estos resultados se han demostrado claramente en la educación primaria y se empiezan a demostrar en los siguientes grados.
- Segundo, si la instrucción se estructura alrededor de problemas cuidadosamente seleccionados, se permite a los estudiantes interactuar durante su solución y se les da la oportunidad de compartir los métodos que usan para resolverlos, se incrementa el desempeño en la resolución de problemas. Debe destacarse que con estos logros no se disminuye el desempeño de las habilidades y conceptos evaluados mediante pruebas estandarizadas.

La investigación ha demostrado también que cuando los estudiantes tienen oportunidades para desarrollar sus propios métodos de solución, son más aptos para aplicar los conocimientos matemáticos en situaciones que conllevan problemas nuevos.

En el aula

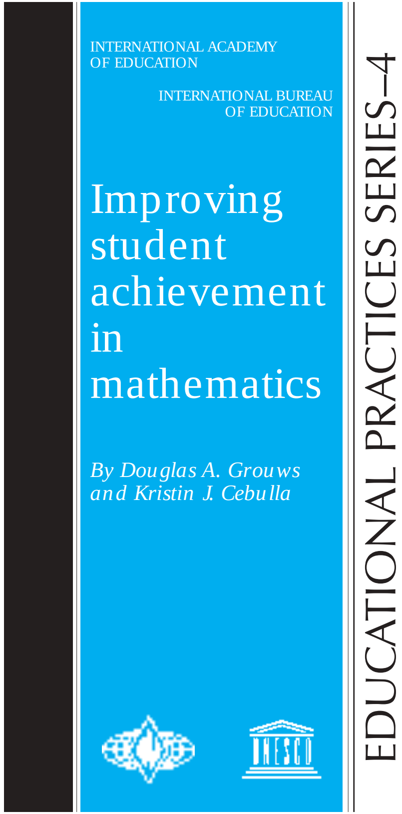
Los resultados de la investigación sugieren que los maestros deberían concentrarse en proporcionar a los estudiantes oportunidades para interactuar en situaciones altamente problemáticas. Además, los maestros deberían alentar a sus estudiantes a encontrar sus propios métodos de solución y propiciar la ocasión para que compartan y comparen sus métodos y resultados. Un modo de organizar ese tipo de enseñanza es que los estudiantes trabajen primero en grupos pequeños y después compartan ideas y soluciones discutiéndolas en clase.

Una técnica de enseñanza útil consiste en que el maestro asigne a sus estudiantes un problema interesante y circule por el aula detectando qué estudiantes están usando tal o cual estrategia (tomando notas si es necesario). En una situación de clase con todo el grupo, el maestro puede hacer que sus estudiantes discutan sus procedimientos para la solución de problemas en un orden cuidadosamente predeterminado, jerarquizando los métodos del más básico al más formal o sofisticado. En Japón, esta estructura de enseñanza ha tenido éxito en muchas lecciones de matemáticas.

Referencias: Boaler (1998); Carpenter *et al.* (1988, 1989, 1998); Cobb, Yackel y Wood (1992); Cobb *et al.* (1991); Cognition and Technology Group (1997); Fennema, Carpenter y Peterson (1989); Fennema *et al.* (1993, 1996); Hiebert y Wearne (1993, 1996); Kamii (1985, 1989, 1994); Stigler y Hiebert (1997); Stigler *et al.* (1999); Wood, Cobb y Yackel (1995); Wood *et al.* (1993), y Yackel, Cobb y Wood (1991).

UNESCO (2000)

International Academy Of Education



5. Openness to student solution methods and student interaction

Teaching that incorporates students' intuitive solution methods can increase student learning, especially when combined with opportunities for student interaction and discussion.

Research findings

Recent results from the TIMSS video study have shown that Japanese classrooms use student solution methods extensively during instruction. Interestingly, the same teaching technique appears in many successful American research projects. Findings from American studies clearly demonstrate two important principles that are associated with the development of students' deep conceptual understanding of mathematics. First, student achievement and understanding are significantly improved when teachers are aware of how students construct knowledge, are familiar with the intuitive solution methods that students use when they solve problems, and utilize this knowledge when planning and conducting instruction in mathematics. These results have been clearly demonstrated in the primary grades and are beginning to be shown at higher-grade levels.

Second, structuring instruction around carefully chosen problems, allowing students to interact when solving these problems, and then providing opportunities for them to share their solution methods result in increased achievement on problem-solving measures. Importantly, these gains come without a loss of achievement in the skills and concepts measured on standardized achievement tests.

Research has also demonstrated that when students have opportunities to develop their own solution methods, they are better able to apply mathematical knowledge in new problem situations.

In the classroom

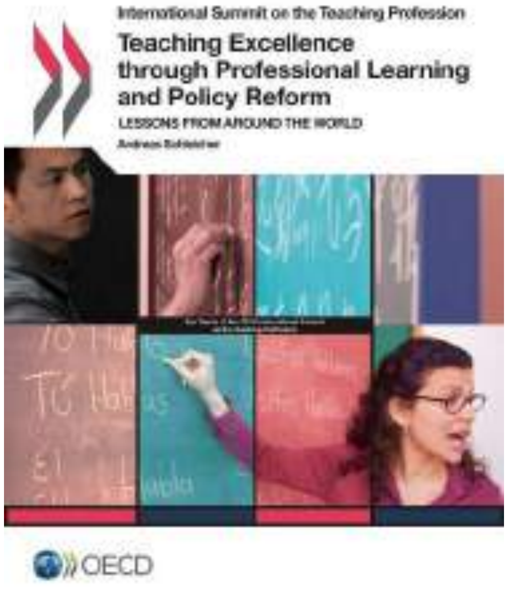
Research results suggest that teachers should concentrate on providing opportunities for students to interact in problem-rich situations. Besides providing appropriate problem-rich situations, teachers must encourage students to find their own solution methods and give them opportunities to share and compare their solution methods and answers. One way to organize such instruction is to have students work in small groups initially and then share ideas and solutions in a whole-class discussion.

One useful teaching technique is for teachers to assign an interesting problem for students to solve and then move about the room as they work, keeping track of which students are using which strategies (taking notes if necessary). In a whole-class setting, the teacher can then call on students to discuss their solution methods in a pre-determined and carefully considered order, these methods often ranging from the most basic to more formal or sophisticated ones. This teaching structure is used successfully in many Japanese mathematics lessons.

References: Boaler, 1998; Carpenter et al., 1988, 1989, 1998; Cobb, Yackel & Wood, 1992; Cobb et al., 1991; Cognition and Technology Group, 1997; Fennema, Carpenter & Peterson, 1989; Fennema et al., 1993, 1996; Hiebert & Wearne, 1993, 1996; Kamii, 1985, 1989, 1994; Stigler & Hiebert, 1997; Stigler et al., 1999; Wood, Cobb & Yackel, 1995; Wood et al., 1993; Yackel, Cobb & Wood, 1991.

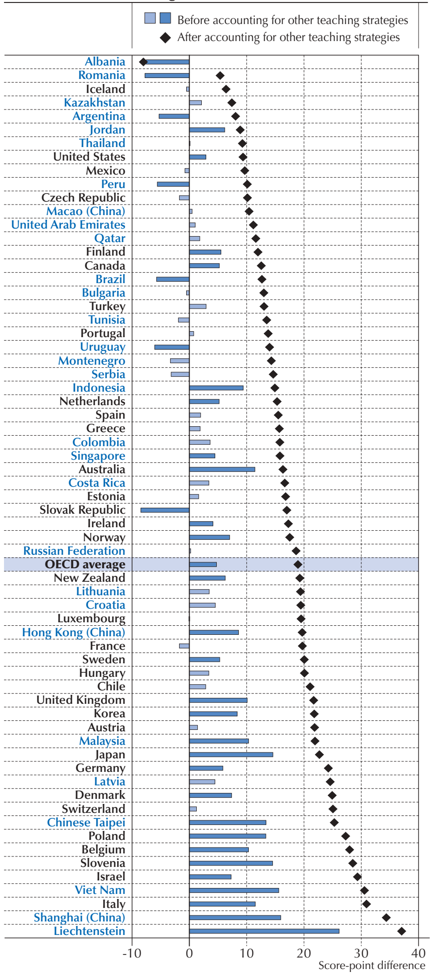
PISA (2016)

TALIS - Teaching Excellence through Professional Learning and Policy Reform



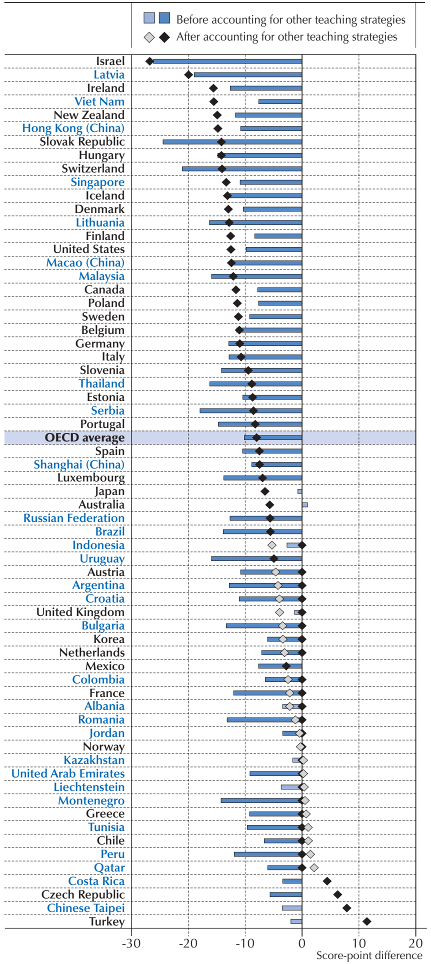
Mathematics performance and cognitive-activation instruction

Score-point difference in mathematics associated with one-unit increase in the index of cognitive-activation instruction



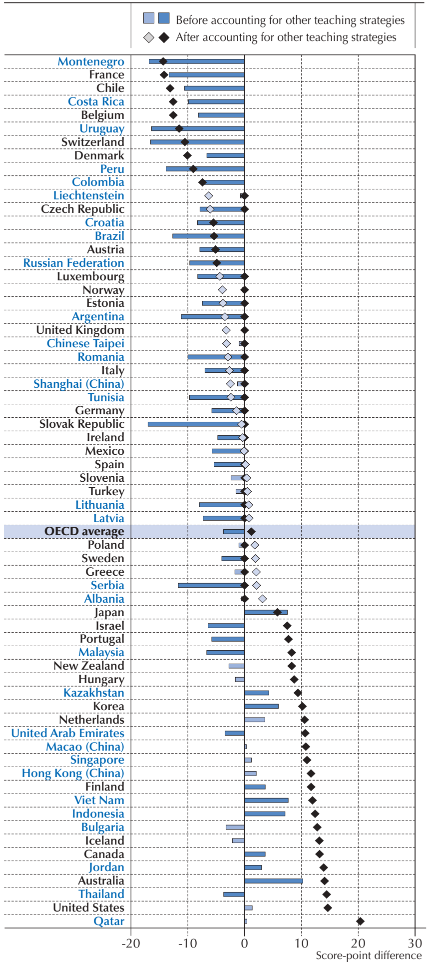
Mathematics performance and formative-assessment instruction

Score-point difference in mathematics associated with one-unit increase in the index of formative-assessment instruction



Mathematics performance and teacher-directed instruction

Score-point difference in mathematics associated with one-unit increase in the index of teacher-directed instruction



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TALIS - Teaching Excellence through Professional Learning and Policy Reform

Instrucción con activación cognitiva:
Según PISA, la instrucción con activación cognitiva se caracteriza por el uso de las siguientes acciones y propuestas de actividades por parte de docente.

El docente:

- pide a los estudiantes que elijan sus propios procedimientos en la solución de problemas complejos.
- propone problemas para los cuales el método de solución no es inmediatamente obvio.
- propone problemas para los cuales los estudiantes necesitan pensar por periodos de tiempo extendido.
- propone problemas en distintos contextos para permitir a los estudiantes saber si han comprendido los conceptos.
- hace preguntas que hace a los estudiantes reflexionar sobre los problemas que propone.
- propone problemas que se pueden solucionar de distintas maneras.
- ayuda a los estudiantes a aprender de los errores que cometen.
- propone problemas que requieren que los estudiantes apliquen lo que han aprendido en distintos contextos.
- solicita explicaciones a los estudiantes acerca de cómo solucionaron los problemas que propone.

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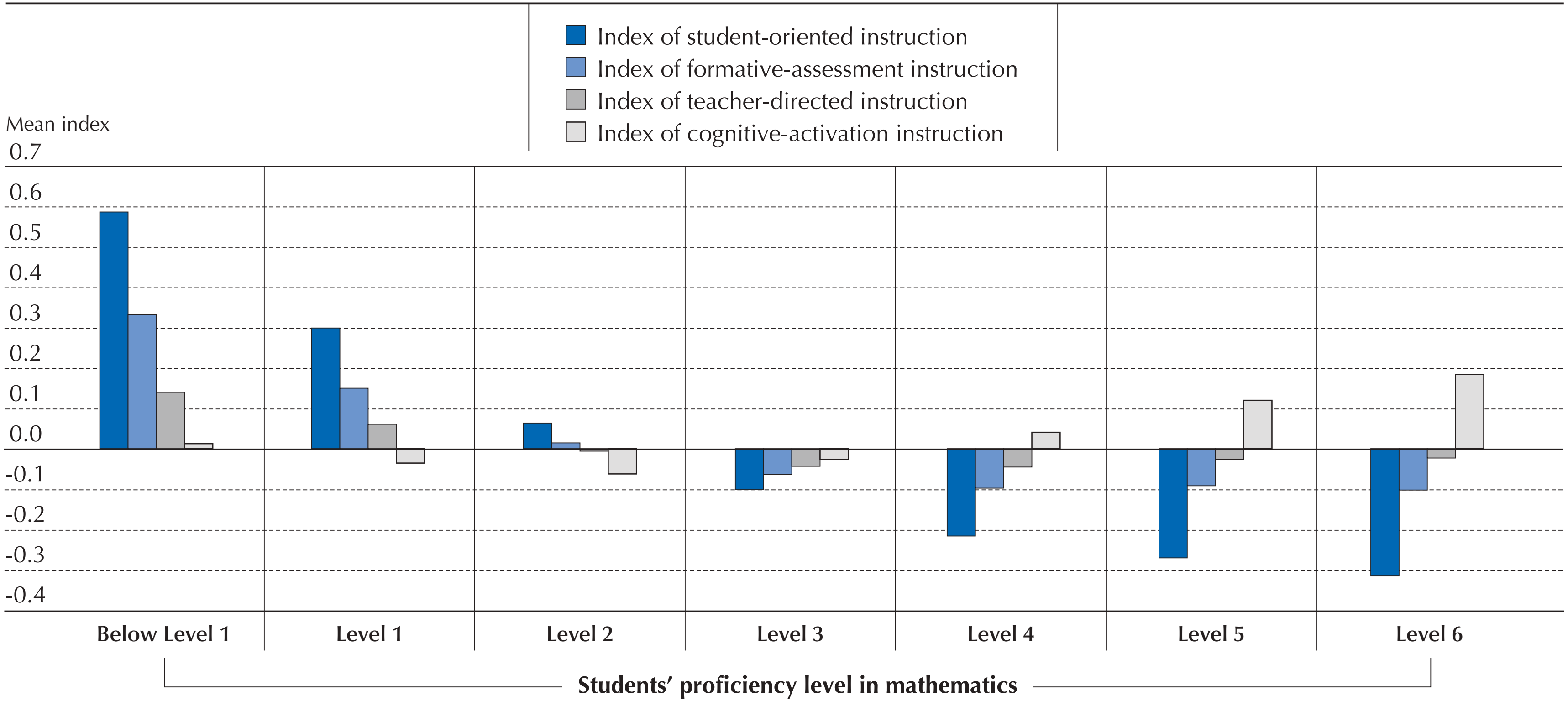
TALIS

Figure 1.5

Teaching strategies, by students' proficiency in mathematics

OECD average



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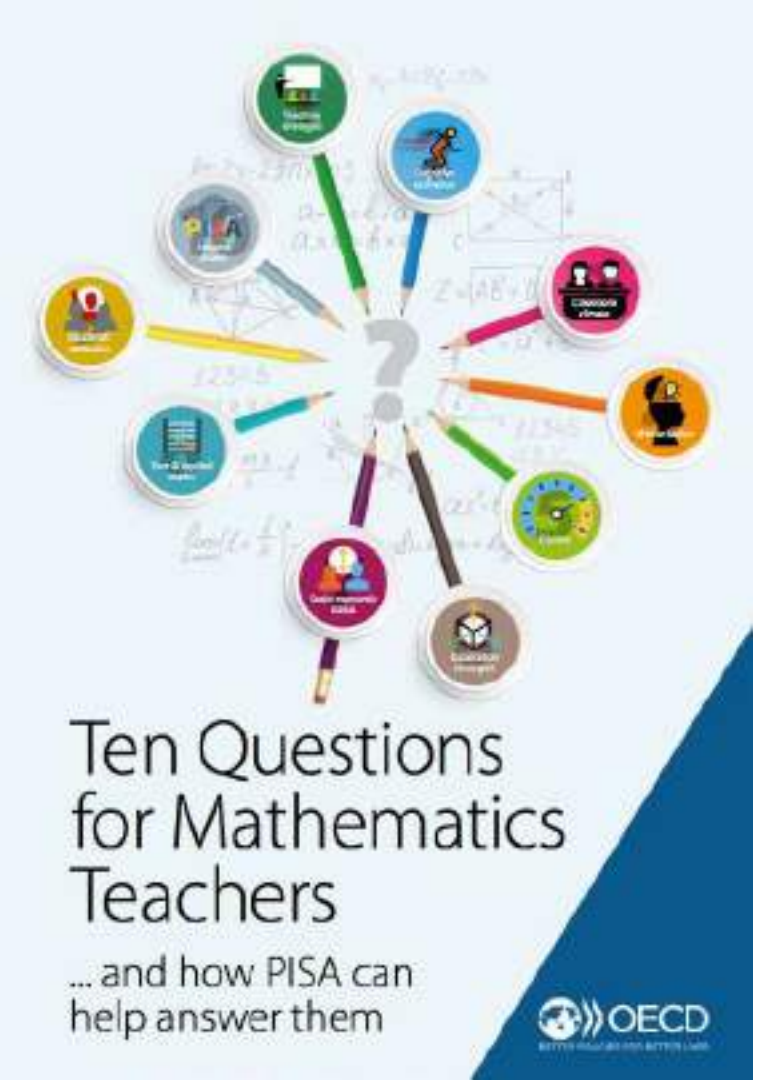
Ten Questions for Mathematics
... and how PISA can help answer them



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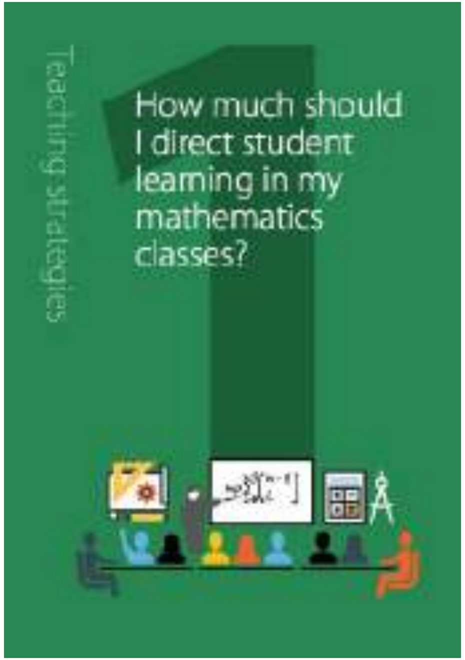
Ten Questions for Mathematics Teachers ... and how PISA can help answer them



1. ¿Cuánto debería dirigir el aprendizaje en mis clases de matemáticas?
2. Como profesor(a) de matemáticas, ¿Qué tan importante es mi relación con los estudiantes?
3. ¿Puedo ayudarle a mis estudiantes a aprender cómo aprender matemáticas?
4. ¿Son algunos métodos de enseñanza de matemáticas que más efectivos que otros?
5. ¿Qué sabemos sobre la relación entre la memorización y el aprendizaje de matemáticas?
6. ¿Debería motivar a mis estudiantes a usar su creatividad en mis clases de matemáticas?
7. ¿Mi enseñanza debería enfatizar conceptos matemáticos, o cómo estos son aplicados en el mundo real?
8. ¿El entorno socio-económico afecta la forma que los estudiantes aprenden matemáticas?
9. ¿Debería estar preocupado por las actitudes que tienen mis estudiantes hacia las matemáticas?
10. ¿Que pueden los profesores aprender de PISA?

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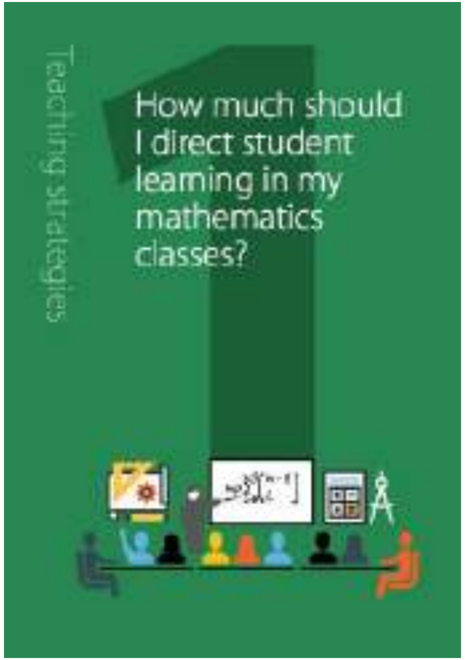
Ten Questions for Mathematics Teachers ... and how PISA can help answer them



1. How much should I direct student learning in my mathematics classes?
2. As a mathematics teacher, how important is the relationship I have with my students?
3. Can I help my students learn how to learn mathematics?
4. Are some mathematics teaching methods more effective than others?
5. What do we know about memorization and learning mathematics?
6. Should I encourage students to use their creativity in mathematics?
7. Should my teaching emphasize mathematical concepts or how those concepts are applied in the real world?
8. Do students' backgrounds influence how they learn mathematics?
9. Should I be concerned about my students' attitudes towards mathematics?
10. What can teachers learn from PISA?

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Ten Questions for Mathematics Teachers ... and how PISA can help answer them

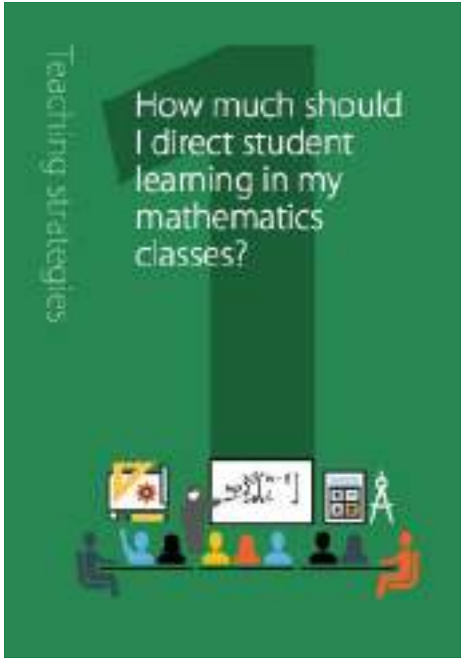
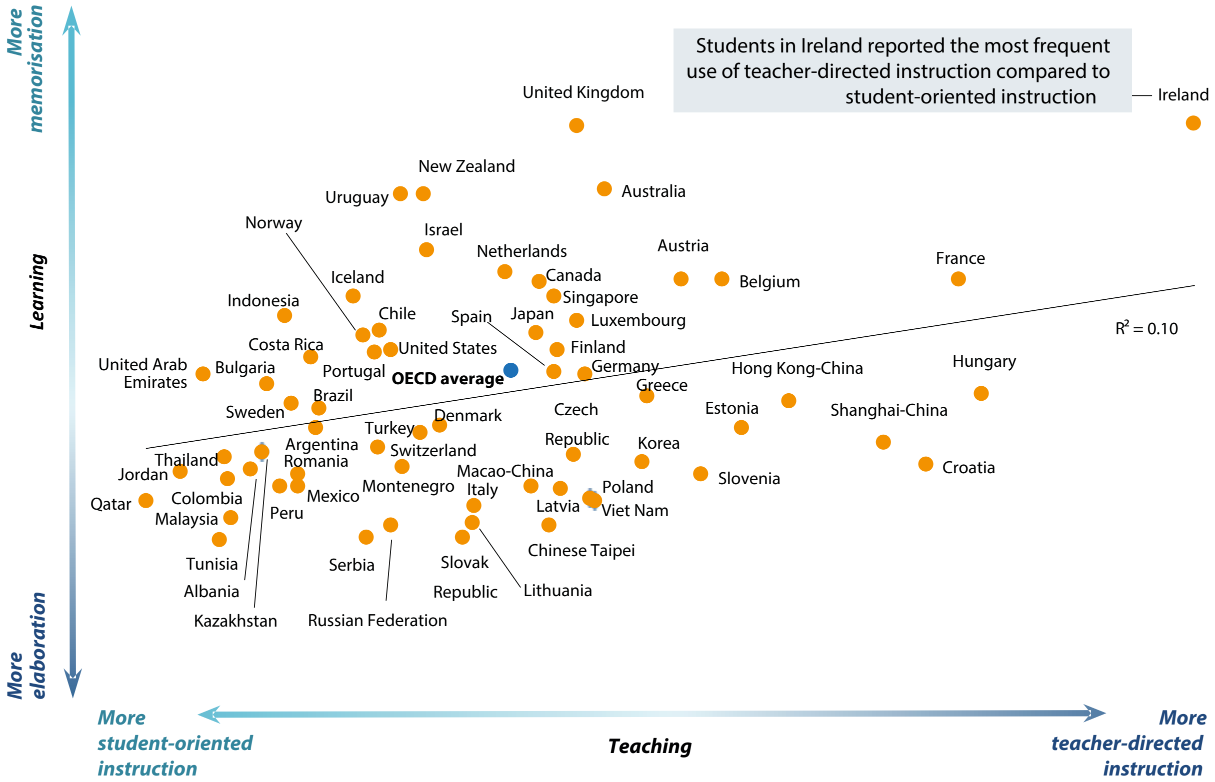


Figure 1.2 **How teachers teach and students learn**

Results based on students' reports



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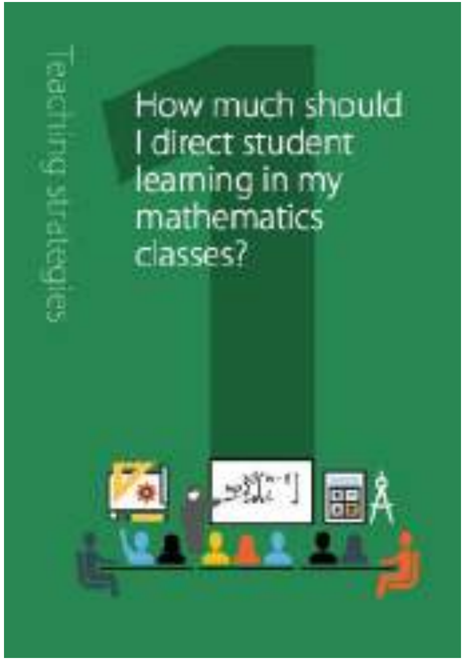
Ten Questions for Mathematics Teachers ... and how PISA can help answer them



WHICH TEACHERS USE ACTIVE-LEARNING TEACHING PRACTICES IN MATHEMATICS?

The TALIS study asked mathematics teachers in eight countries about their regular teaching practices. The study included four active-learning teaching practices that overlap in large part with student-oriented practices: placing students in small groups, encouraging students to evaluate their own progress, assigning students long projects, and using ICT for class work. These practices have been shown by many research studies to have positive effects on student learning and motivation. TALIS data show that teachers who are confident in their own abilities are more likely to engage in active-teaching practices. This is a somewhat logical finding, as active practices could be thought of as more "risky" than direct-teaching methods. It can be challenging to use ICT in your teaching or have students work in groups if you are not confident that you have the skills needed in pedagogy, content or classroom management.

Figure 1.3 **How teachers' self-efficacy is related to the use of active-learning instruction**

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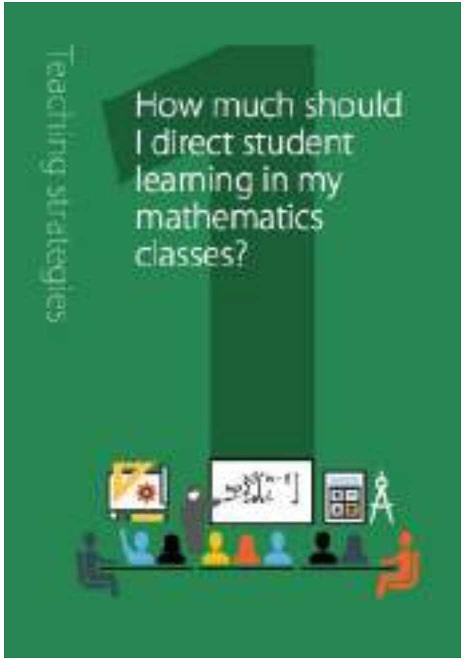
Ten Questions for Mathematics Teachers ... and how PISA can help answer them

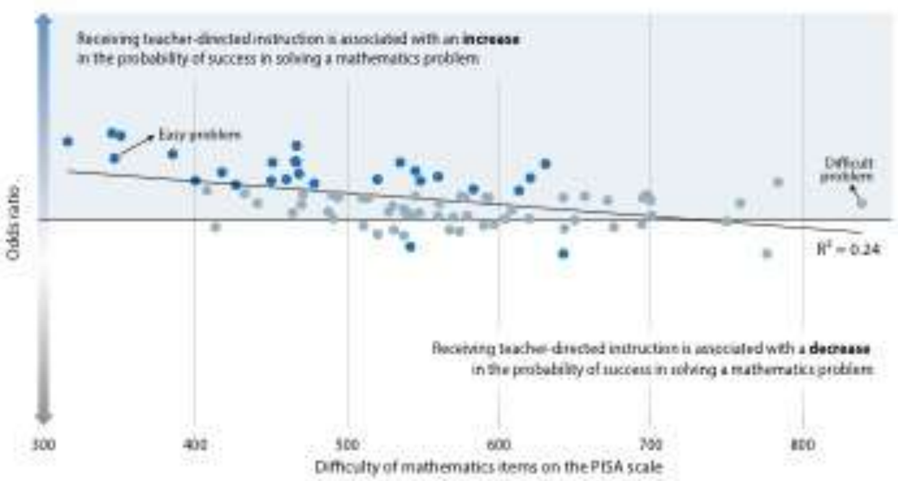


HOW CAN A VARIETY OF TEACHING STRATEGIES BENEFIT STUDENT ACHIEVEMENT?

When looking at students' mean mathematics scores on the PISA assessment alongside their exposure to the teaching strategies discussed in this chapter, another reason for using a variety of teaching strategies emerges. Let's look first at the most commonly used teaching practices in mathematics, teacher-directed strategies. The data indicate that when teachers direct student learning, students are slightly more likely to be successful in solving the easiest mathematics problems in PISA. Yet as the problems become more difficult, students with more exposure to direct instruction no longer have a better chance of success. Figure 1.4 shows the relationship between the use of teacher-directed strategies and students' success on mathematics problems of varying difficulty.

Figure 1.4 **Teacher-directed instruction and item difficulty**

Odds ratio, after accounting for other teaching strategies, OECD average



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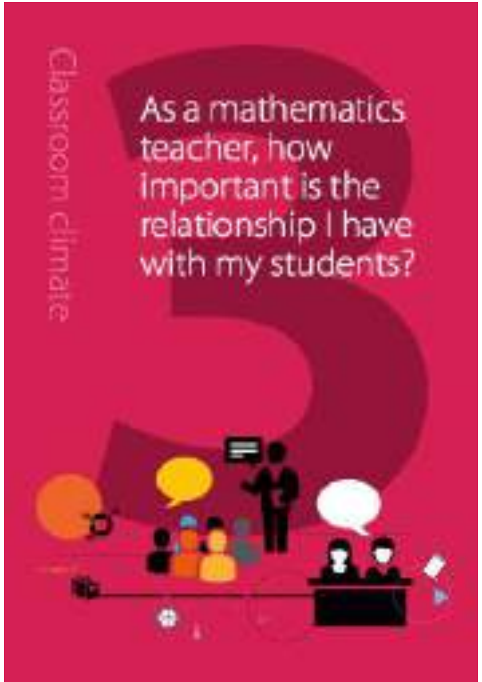
Ten Questions for Mathematics Teachers ... and how PISA can help answer them



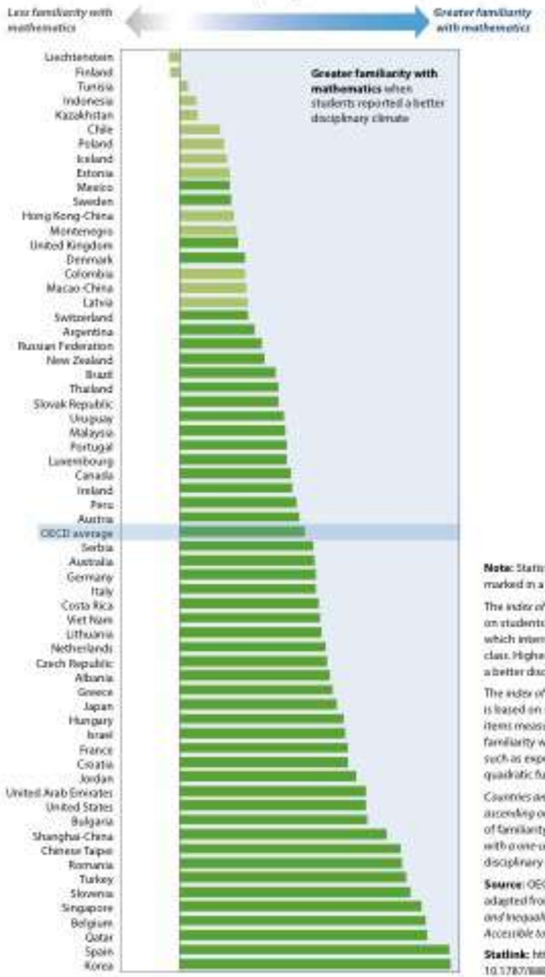
Every teacher has great teaching days. These are the days when your lesson works, and the students are motivated to learn and are engaged in class activities. Think back to your last great teaching day: how was the learning environment in your classroom? Did you continually have to discipline students because of their behaviour? Were students late for class or causing other disruptions? Or were learners staying on task, actively participating and treating you and their peers with respect? This kind of positive classroom climate, with minimal interference, gives teachers more time to spend on teaching, and makes those great teaching days possible. Teachers don't have to spend time addressing disruptions, and the classroom becomes an environment in which learning can take place. What's more, the quality of the learning environment is not only related to how teachers are able to teach, but also how they feel about their jobs and their own abilities as teachers.

WHAT IS A GOOD CLASSROOM ENVIRONMENT FOR MATHEMATICS TEACHING AND LEARNING?

A positive classroom climate, good classroom management and strong relationships between teachers and learners should be considered prerequisites for high-quality teaching. In general, more teaching, and presumably learning, occurs when there is a positive school environment, including support from teachers and good classroom management. In addition, the disciplinary climate of the classroom is related to what and how teachers are able to teach. For example, it might be easier for teachers to use cognitive-activation strategies, such as encouraging students to be reflective in their thinking, in classrooms where students stay on task and disruptions are kept to a minimum.

Figure 3.1 **Disciplinary climate and familiarity with mathematics**

Change in students' familiarity with mathematics associated with a better disciplinary climate in class



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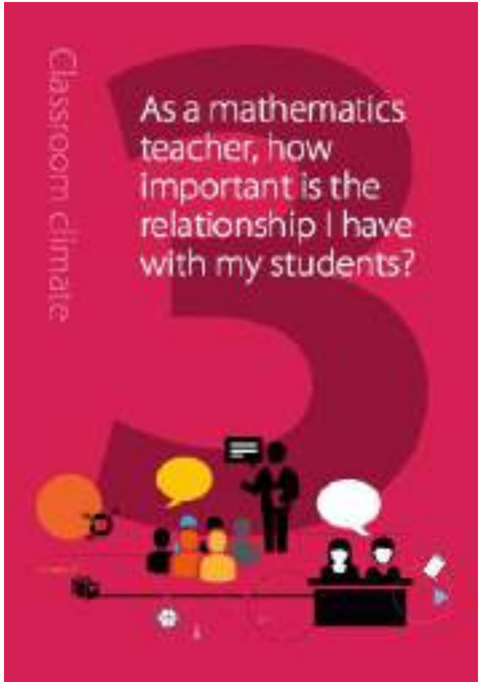
Ten Questions for Mathematics Teachers ... and how PISA can help answer them



WHAT CAN TEACHERS DO?

Focus time and energy on creating a positive classroom climate. If classroom management and discipline are of particular concern to you, find a way to get additional support. Speak to or observe other teachers in your school to learn successful classroom-management strategies. Ask your school leadership if you can look for ongoing professional development on this issue.

Invest time in building strong relationships with your students. This is particularly demanding for those teachers who see upwards of 150 students each day, but it could make a difference to both your students' learning and your teaching – not to mention your own well-being as a teacher. Students want to feel that their teachers treat them fairly, listen to them and will continue teaching them until they understand the material. In addition, learning about students' lives outside of school might help you to connect topics in mathematics with real-world situations that are meaningful to your students.

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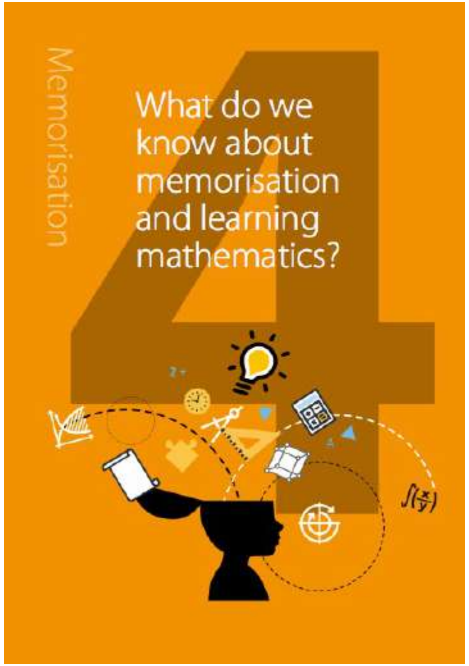
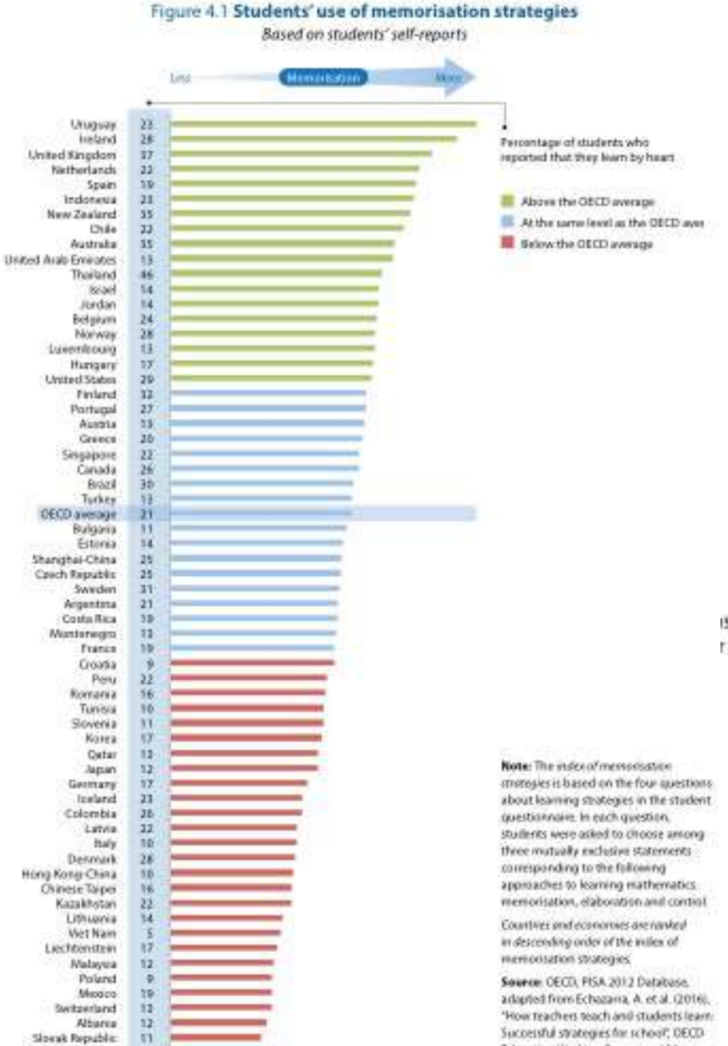
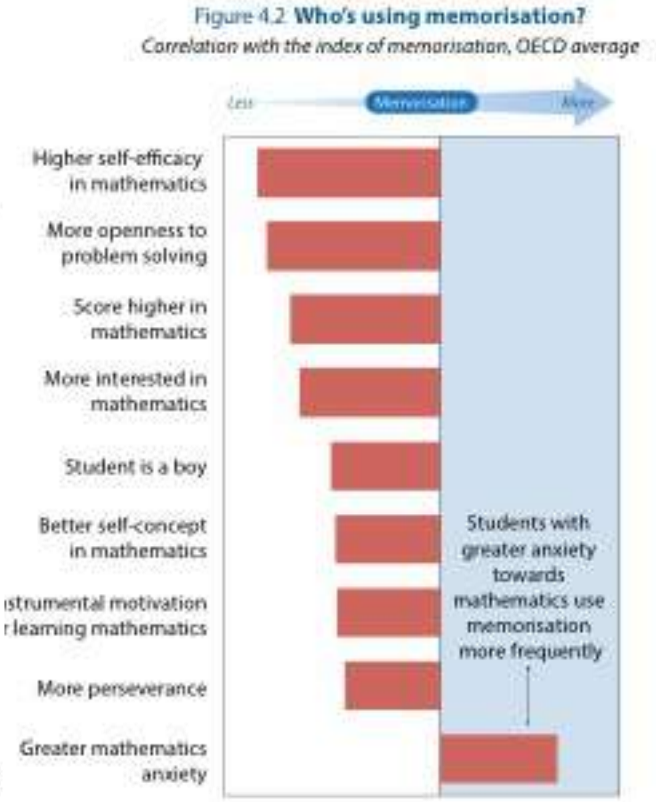
Ten Questions for Mathematics Teachers ... and how PISA can help answer them







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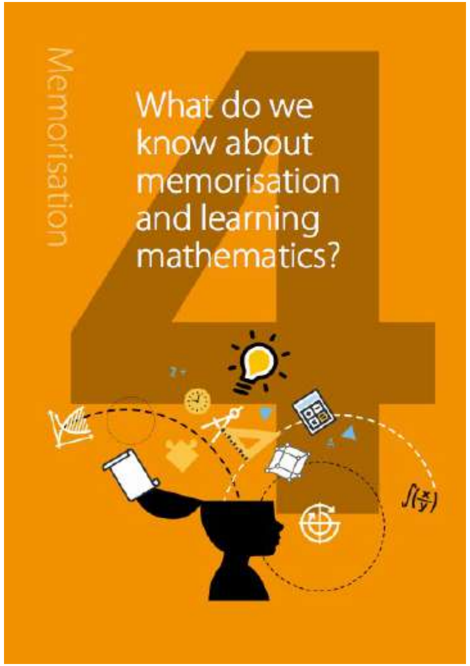
Ten Questions for Mathematics Teachers ... and how PISA can help answer them



WILL MEMORISATION HELP OR HURT MY STUDENTS' PERFORMANCE IN MATHEMATICS?

Some experts in mathematics education consider memorisation to be an elementary strategy that is better suited to solving routine problems that require only a shallow understanding of mathematics concepts.[2] PISA results reinforce this view. They show that students who reported using memorisation strategies are indeed successful on easier mathematics tasks. For example, one of the easiest mathematics problems in the PISA 2012 assessment was a multiple-choice question involving a simple bar chart. Some 87% of students across PISA-participating education systems answered this question correctly. Students who reported that they use memorisation strategies to learn mathematics had about the same success rate on this easy item as students who reported using other learning strategies.

Although memorisation seems to work for the easiest mathematics problems, its success as a learning strategy does not extend much beyond that. According to the data, as problems become more challenging, students who use memorisation are less likely to be able to solve them correctly. Results are even worse for the most challenging mathematics problems. Only 3% of students answered the most difficult question on the 2012 PISA exam correctly. Solving this problem required multiple

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Ten Questions for Mathematics Teachers ... and how PISA can help answer them

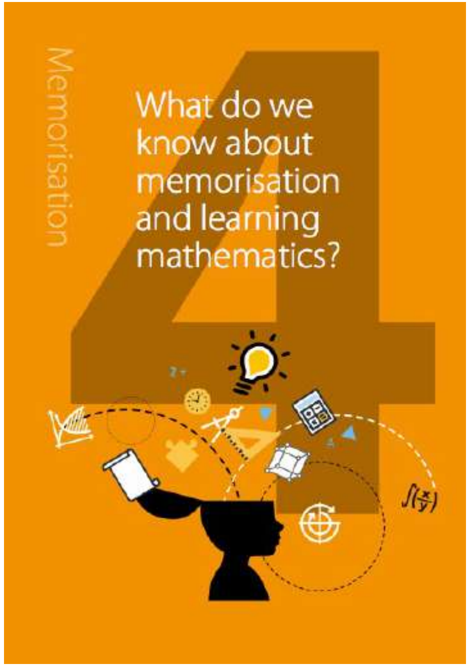
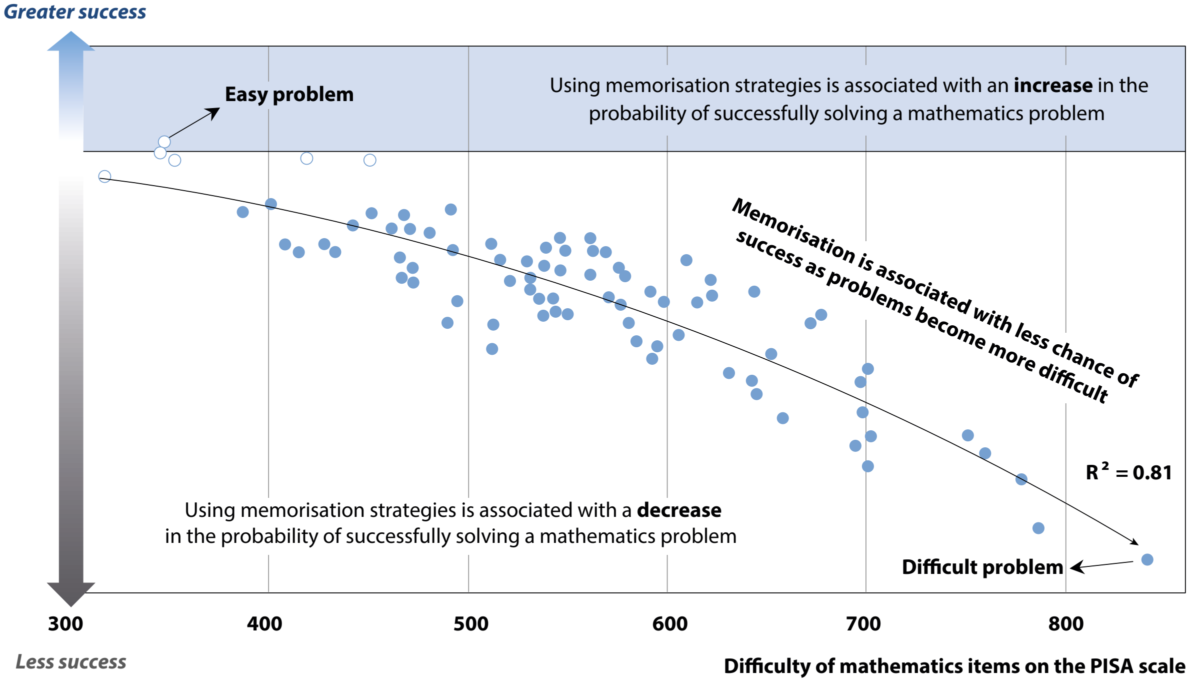


Figure 4.3 **Memorisation strategies and item difficulty**

Odds ratio across 48 education systems



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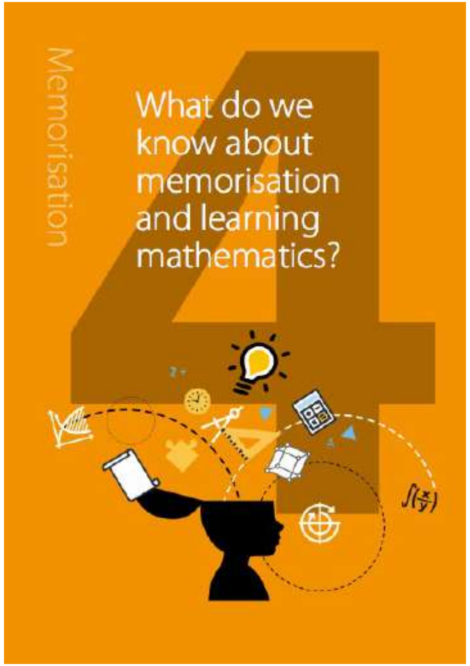
Ten Questions for Mathematics Teachers ... and how PISA can help answer them



WHAT CAN TEACHERS DO?

Encourage students to complement memorisation with other learning strategies. Memorisation can be used for some tasks in mathematics, such as recalling formulas or automating simple calculations to speed up problem solving. This will help students free up time for deeper thinking as they encounter more difficult problems later on. However, you should encourage your students to go beyond memorisation if you want them to understand mathematics, and solve real complex problems later in life.

Use memorisation strategies to build familiarity and confidence. Students may practice or repeat certain procedures as this helps consolidate their understanding of concepts and builds familiarity with problem-solving approaches. These activities don't have to be boring; teachers can find free interactive software or games on line to make such practice activities more interesting to students.

Notice how your students learn. Learners who are less confident in their own mathematical abilities or more prone to anxiety may rely too much on memorisation. Urge those students to use other learning strategies as well by helping them make connections between concepts and real-world problems and encouraging them to set their own goals for learning mathematics. Also, remember that the way you teach concepts and assess students' understanding can influence how students approach mathematics.

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Ten Questions for Mathematics Teachers ... and how PISA can help answer them

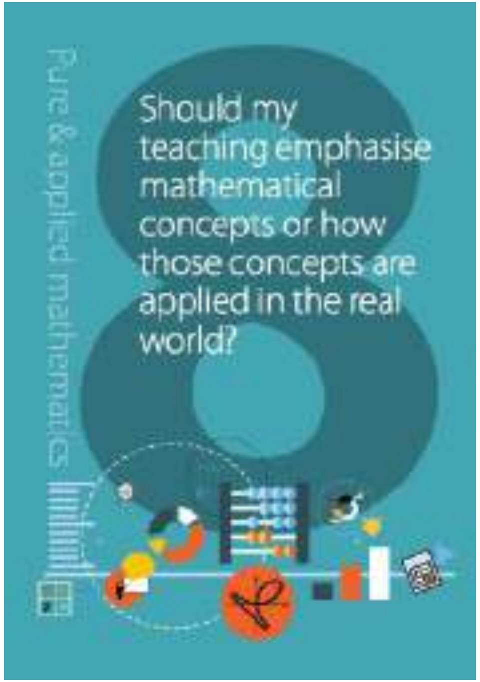
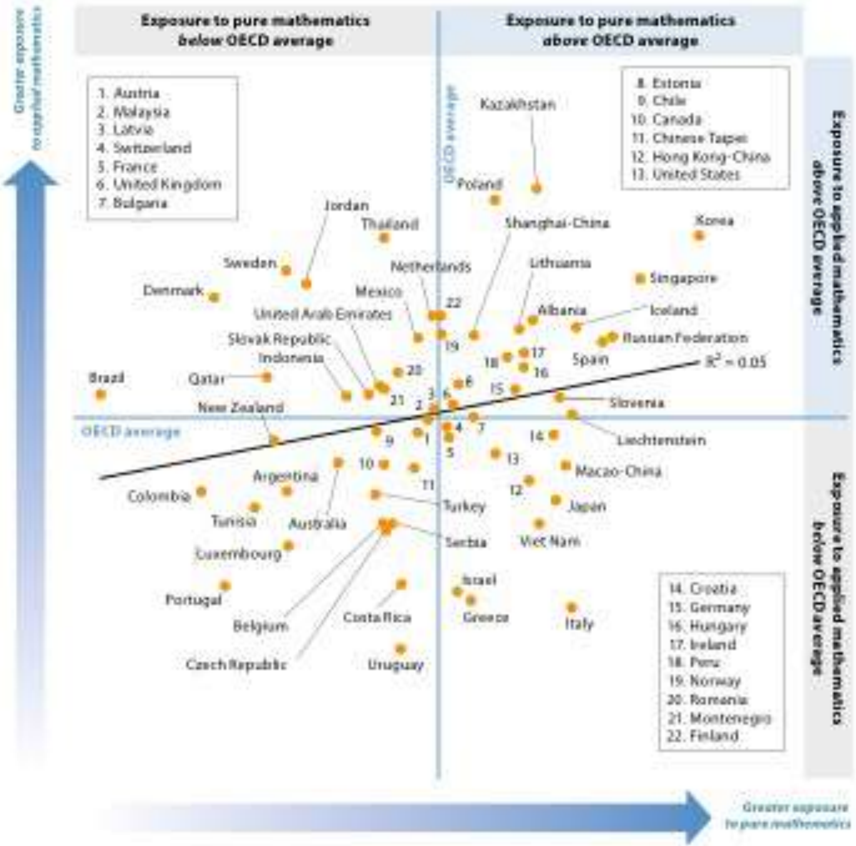


Figure 8.1 **Relationship between students' exposure to pure and applied mathematics, by country**



Notes: The *index of exposure to pure mathematics* measures student-reported experience with mathematics tasks requiring knowledge of algebra (linear and quadratic equations).

The *index of exposure to applied mathematics* measures student-reported experience with applied mathematical tasks at school, such as working out from a train timetable how long it would take to get from one place to another or calculating how much more expensive a computer would be after adding tax.

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Ten Questions for Mathematics Teachers ... and how PISA can help answer them

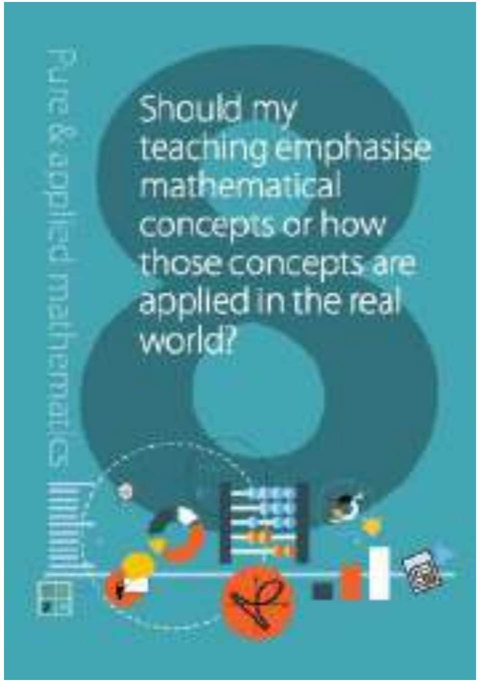
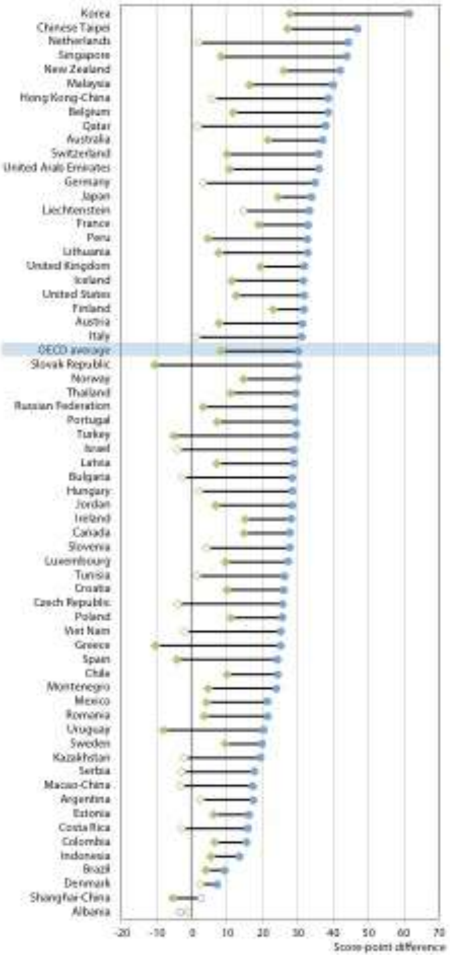


Figure 8.2 **Relationship between exposure to pure mathematics and mathematics performance**

Score-point difference in mathematics performance associated with greater exposure to pure or applied mathematics



Exposure to pure mathematics
Exposure to applied mathematics

Notes: Statistically significant values are marked in a darker tone.

The *index of exposure to pure mathematics* measures student-reported experience with mathematics tasks at school requiring knowledge of algebra (linear and quadratic equations).

The *index of exposure to applied mathematics* measures student-reported experience with applied mathematical tasks at school, such as working out from a train timetable how long it would take to get from one place to another or calculating how much more expensive a computer would be after adding tax.

Countries and economies are ranked in descending order of the score-point difference associated with a one-unit increase in the index of exposure to pure mathematics.

Source: OECD, PISA 2012 Database, adaptated from OECD (2016), "Equations and Inequalities: Making Mathematics Accessible to All", OECD Publishing, Paris.

StatLink: http://dx.doi.org/10.1787/888933414925

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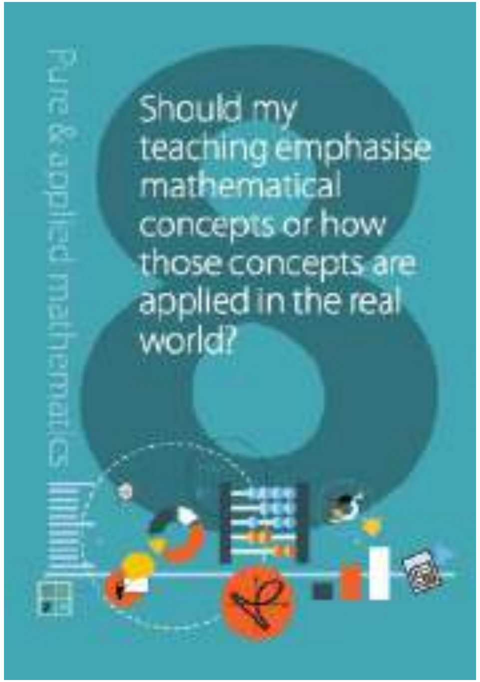
Ten Questions for Mathematics Teachers ... and how PISA can help answer them



WHAT DOES THIS MEAN FOR MY TEACHING?

Knowledge of mathematics terminology, facts and procedures is beneficial for performance on mathematics tasks in general, and especially useful for more challenging problems. But it takes more than content knowledge and practice to be successful at solving problems. Students still need to be able to think and reason mathematically. PISA analyses looked at two difficult problems from the 2012 assessment, one that required students to answer a question using a specific formula (DRIP RATE Question 1) and one that asked students to engage in complex reasoning using a formula that they should know but that is not referred to in the text (REVOLVING DOOR Question 2). The second question required students to be able to model a real situation in mathematical form, which requires a high level of skill in mathematics (see Box 8.1 on the following page for the full text of both problems).

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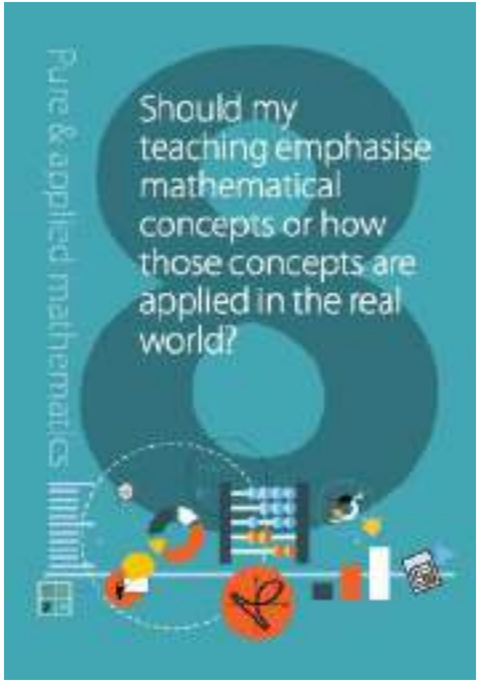
Ten Questions for Mathematics Teachers ... and how PISA can help answer them



WHAT CAN TEACHERS DO?

Cover core mathematics ideas in sufficient depth and show how they are related. Students often don't understand how the mathematics they are learning in school might be used in the real world. In addition, the order of topics presented in many mathematics textbooks doesn't make it clear how certain concepts are related to each other. Work with colleagues in your department to teach the curriculum in a way that makes these connections clearer for students. When students understand the relationships among the topics, they stop seeing mathematics as a laundry-list of formulas to memorise, and start to make sense of what they learn. In addition, when students understand why concepts are important for their future life or possible careers, they might become more interested in mathematics.

Don't just cover the fundamentals of the curriculum. Teachers should of course cover the fundamental elements of the mathematics curriculum but still find time to expose students to problems that promote conceptual understanding and activate their cognitive abilities. To do this, it might be worthwhile to increase your use of problem solving as a method of teaching mathematics. Problem solving can be used to introduce core mathematical concepts through lessons involving exploration and discovery. It will prepare students for some of the more complex reasoning that is involved in more difficult mathematics problems.

Provide students with a variety of applied problems to solve. Teaching today's mathematics curricula, which are thought to be general, often makes it challenging for students to apply this knowledge to concrete problems. Students need to be exposed to several different representations of concepts in order to develop the skills needed to translate between the real world and world of mathematics, and vice versa. Give students a variety of problems that includes contextualised problems in which students need to apply knowledge to find a solution to a problem encountered in everyday life. Pedagogies such as project- or problem-based learning present students with real-world problems that they have to solve, often as a team, applying the skills they have just learned.

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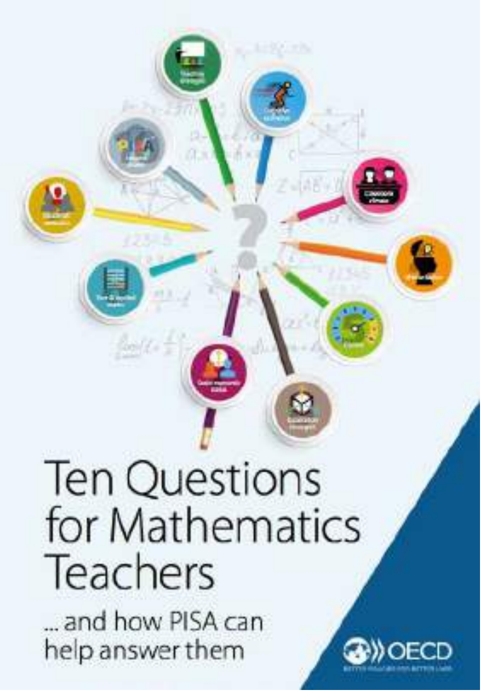
Ten Questions for Mathematics Teachers ... and how PISA can help answer them



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2. Como profesor(a) de matemáticas, ¿Qué tan importante es mi relación con los estudiantes?
3. ¿Puedo ayudarle a mis estudiantes a aprender cómo aprender matemáticas?
4. ¿Son algunos métodos de enseñanza de matemáticas que más efectivos que otros?
5. ¿Qué sabemos sobre la relación entre la memorización y el aprendizaje de matemáticas?
6. ¿Debería motivar a mis estudiantes a usar su creatividad en mis clases de matemáticas?
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8. ¿El entorno socio-económico afecta la forma que los estudiantes aprenden matemáticas?
9. ¿Debería estar preocupado por las actitudes que tienen mis estudiantes hacia las matemáticas?
10. ¿Que pueden los profesores aprender de PISA?

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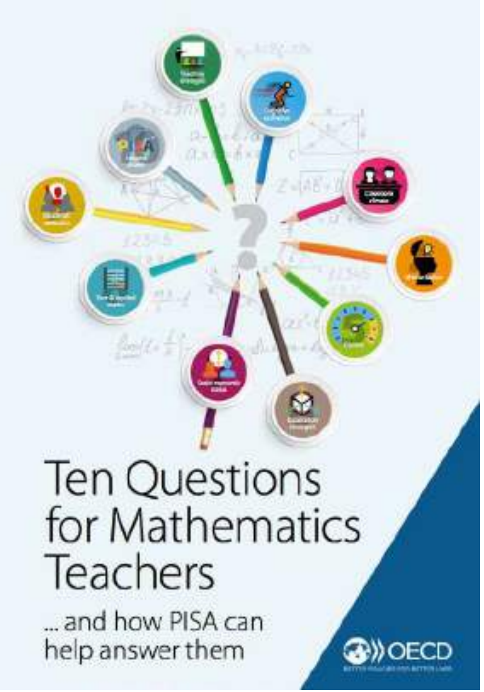
Ten Questions for Mathematics Teachers ... and how PISA can help answer them



1. How much should I direct student learning in my mathematics classes?
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8. What can teachers learn from PISA?
9. Do students' backgrounds influence how they learn mathematics?
10. Should I be concerned about my students' attitudes towards mathematics?

Schoenfeld (2015)

The Teaching for Robust Understanding (TRU) Framework

… for characterizing powerful learning environments in crisp and actionable ways. It provides a straightforward and accessible language for discussing what happens (and should happen) in classrooms, in professional preparation and professional Development (PD), and, potentially, in every context where learning happens.

The Five Dimensions of Powerful Classrooms

The Content	Cognitive Demand	Equitable Access to Content	Agency, Authority, and Identity	Uses of Assessment
The extent to which the content students engage with represents our best current disciplinary understandings (as in CCSS, NGSS, etc.). Students should have opportunities to learn important content and practices, and to develop productive disciplinary habits of mind.	*The extent to which classroom interactions create and maintain an environment of productive intellectual challenge conducive to students' disciplinary development. There is a happy medium between spoon-feeding content in bite-sized pieces and having the challenges so large that students are lost at sea.*	*The extent to which classroom activity structures invite and support the active engagement of all of the students in the classroom with the core content being addressed by the class. No matter how rich the content being discussed, a classroom in which a small number of students get most of the "air time" is not equitable.*	*The extent to which students have opportunities to "walk the walk and talk the talk," building on each other's ideas, in ways that contribute to their development of agency (the capacity and willingness to engage) and authority (recognition for being a good thinker), resulting in positive identities as thinkers and learners.*	*The extent to which the teacher solicits student thinking and subsequent instruction responds to those ideas, by building on productive beginnings or addressing emerging misunderstandings. Powerful instruction "meets students where they are" and gives them opportunities to move forward.*

Schoenfeld (2015)

The Teaching for Robust Understanding (TRU) Framework

Rúbrica de observación

	The Mathematics	Cognitive Demand	Access to Mathematical Content	Agency, Authority, and Identity	Formative Assessment
	How accurate, coherent, and well justified is the mathematical content?	*To what extent are students supported in grappling with and making sense of mathematical concepts?*	*To what extent does the teacher support access to the content of the lesson for all students?*	*To what extent are students the source of ideas and discussion of them? How are student contributions framed?*	*To what extent is students' mathematical thinking surfaced; to what extent does instruction build on student ideas when potentially valuable or address misunderstandings when they arise?*
1	Classroom activities are unfocused or skills-oriented, lacking opportunities for engagement in key practices such as reasoning and problem solving.	Classroom activities are structured so that students mostly apply memorized procedures and/or work routine exercises.	There is differential access to or participation in the mathematical content, and no apparent efforts to address this issue.	The teacher initiates conversations. Students' speech turns are short (one sentence or less), and constrained by what the teacher says or does.	Student reasoning is not actively surfaced or pursued. Teacher actions are limited to corrective feedback or encouragement.
2	Activities are primarily skills-oriented, with cursory connections between procedures, concepts and contexts (where appropriate) and minimal attention to key practices.	Classroom activities offer possibilities of conceptual richness or problem solving challenge, but teaching interactions tend to "scaffold away" the challenges, removing opportunities for productive struggle.	There is uneven access or participation but the teacher makes some efforts to provide mathematical access to a wide range of students.	Students have a chance to explain some of their thinking, but "the student proposes, the teacher disposes": in class discussions, student ideas are not explored or built upon.	The teacher refers to student thinking, perhaps even to common mistakes, but specific students' ideas are not built on (when potentially valuable) or used to address challenges (when problematic).
3	Classroom activities support meaningful connections between procedures, concepts and contexts (where appropriate) and provide opportunities for engagement in key practices.	The teacher's hints or scaffolds support students in productive struggle in building understandings and engaging in mathematical practices.	The teacher actively supports and to some degree achieves broad and meaningful mathematical participation; **OR** what appear to be established participation structures result in such engagement.	Students explain their ideas and reasoning. The teacher may ascribe ownership for students' ideas in exposition, **AND/OR** students respond to and build on each other's ideas.	The teacher solicits student thinking and subsequent instruction responds to those ideas, by building on productive beginnings or addressing emerging misunderstandings.

Figure 2. The TRU Math Summary Rubric

Schoefeld (2014)

The Teaching for Robust Understanding (TRU) Framework

Five Dimensions of Powerful Sites for Professional Growth

The Content (Professionalism)	Cognitive Demand (Room to Grow)	Equitable Access to Professional Growth	Agency, Authority, and Identity	Uses of Assessment
The extent to which the adults in the environment are supported in practices that build their capacity to create rich learning environments for their students.	*The extent to which adults at the site are coached and supported in ways that meet them where they are, and help them work on problems of practice that support their growth.*	*The extent to which support and accountability structures enfranchise all adults in the environment and help them grow.*	*The extent to which adults in the environment develop confidence and pride in their accomplishment as professionals, taking increasing responsibility for their growth and performance.*	*The extent to which accountability structures identify strengths and weaknesses, and help to support professional growth.*

Schoenfeld (2015)

The Teaching for Robust Understanding (TRU) Framework

La clase desde el punto de vista del profesor

The Mathematics	• Is it important, coherent, connected? • Opportunities for thinking and problem solving?
Cognitive Demand	• Do students have opportunities for sense making? • Do they engage in productive struggle?
Access and Equity	• Who participates in what ways? • Do *all* students engage in sense-making?
Agency, Ownership	• Do students have the opportunity to do and talk math? • Do they come to see themselves as math people?
Formative Assessment	• Does classroom discussion reveal what students understand, so that instruction may be adapted to help students learn?

Schoenfeld (2015)

The Teaching for Robust Understanding (TRU) Framework

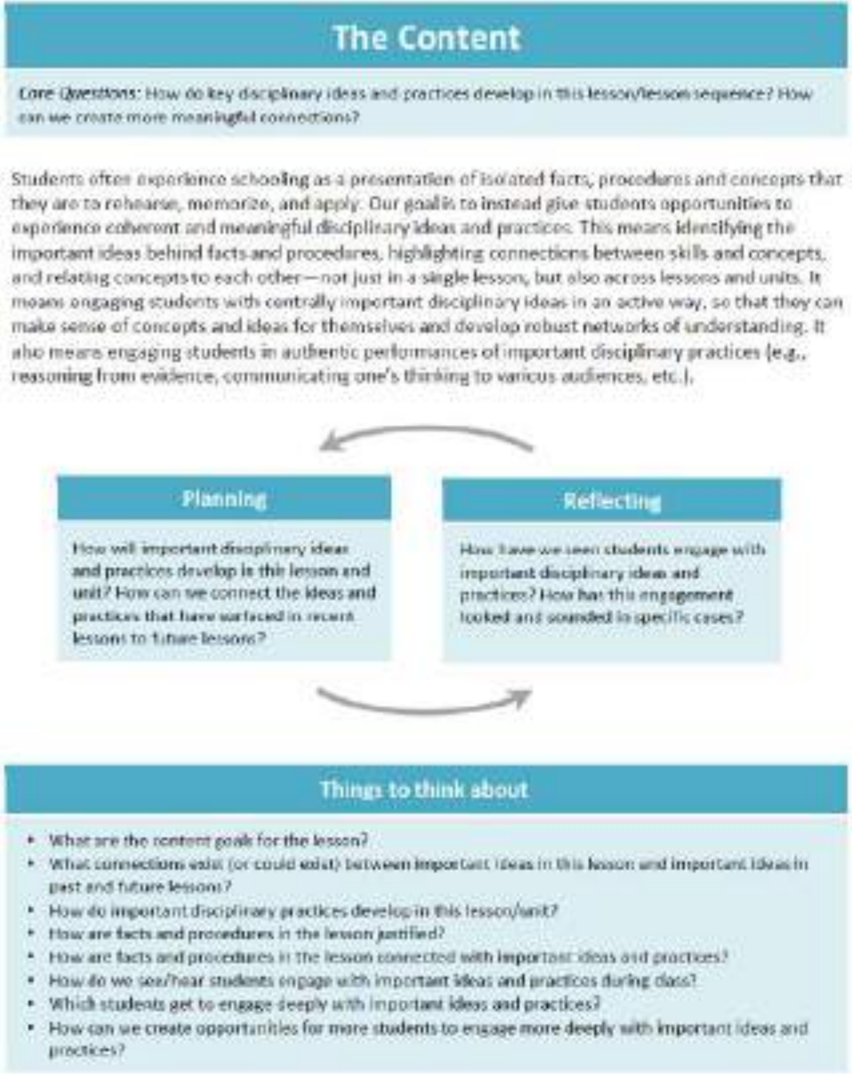
Observe the Lesson Through a Student's Eyes	
The Content	• What's the big idea in this lesson? • How does it connect to what I already know?
Cognitive Demand	• How long am I given to think, and to make sense of things? • What happens when I get stuck? • Am I invited to explain things, or just give answers?
Equitable Access to Content	• Do I get to participate in meaningful math learning? • Can I hide or be ignored? In what ways am I kept engaged?
Agency, Ownership, and Identity	• What opportunities do I have to explain my ideas? In what ways are they built on? • How am I recognized as being capable and able to contribute?
Formative Assessment	• How is my thinking included in classroom discussions? • Does instruction respond to my ideas and help me think more deeply?

La clase desde el punto de vista del estudiante

Schoenfeld (2015)

The Teaching for Robust Understanding (TRU) Framework

Planeación y reflexión



The Content

Core Questions: How do key disciplinary ideas and practices develop in this lesson/lesson sequence? How can we create more meaningful connections?

Students often experience schooling as a presentation of isolated facts, procedures and concepts that they are to rehearse, memorize, and apply. Our goal is to instead give students opportunities to experience coherent and meaningful disciplinary ideas and practices. This means identifying the important ideas behind facts and procedures, highlighting connections between skills and concepts, and relating concepts to each other—not just in a single lesson, but also across lessons and units. It means engaging students with centrally important disciplinary ideas in an active way, so that they can make sense of concepts and ideas for themselves and develop robust networks of understanding. It also means engaging students in authentic performances of important disciplinary practices (e.g., reasoning from evidence, communicating one's thinking to various audiences, etc.).

Planning

How will important disciplinary ideas and practices develop in this lesson and unit? How can we connect the ideas and practices that have surfaced in recent lessons to future lessons?

Reflecting

How have we seen students engage with important disciplinary ideas and practices? How has this engagement looked and sounded in specific cases?

Things to think about

- What are the content goals for the lesson?
- What connections exist (or could exist) between important ideas in this lesson and important ideas in past and future lessons?
- How do important disciplinary practices develop in this lesson/unit?
- How are facts and procedures in the lesson justified?
- How are facts and procedures in the lesson connected with important ideas and practices?
- How do we see/hear students engage with important ideas and practices during class?
- Which students get to engage deeply with important ideas and practices?
- How can we create opportunities for more students to engage more deeply with important ideas and practices?

Deborah Ball (2000)

Mathematical

Knowledge for Teaching

The knowledge and skills might be captured with practices and dispositions such as these:

Representing and connecting representations (e.g., symbols, graphs, geometric models)
Teaching requires being able to represent ideas and connect carefully across different representations – symbolic, graphical, and geometric. Representation is a central feature of the work of teaching; skill and sensibilities with representing particular ideas or procedures is as fundamental as knowing their definitions.

Mathematical language and definitions
Using mathematical language with care, and understanding how definitions and precision shape mathematical problem solving and thinking is another element crucial to understanding how teachers must use — and therefore know — mathematics.

Mathematical reasoning and justification

Good sense about mathematical precision

Mathematical curiosity and interest
Teachers need to be people who are themselves curious and interested in mathematics and who are fascinated by students' mathematical curiosities and interests.

Deborah Ball (2000)

Mathematical

Knowledge for Teaching

The mathematics that teachers have to do in the course of their work:

- Design mathematically accurate explanations that are comprehensible and useful for students
- Use mathematically appropriate and comprehensible definitions
- Represent ideas carefully, mapping between a physical or graphical model, the symbolic notation, and the operation or process
- Interpret and make mathematical and pedagogical judgments about students’ questions, solutions, problems, and insights (both predictable and unusual)
- Be able to respond productively to students’ mathematical questions and curiosities
- Make judgments about the mathematical quality of instructional materials and modify as necessary
- Be able to pose good mathematical questions and problems that are productive for students’ learning
- Assess students’ mathematics learning and take next steps

Deborah Ball (2000)

Mathematical Knowledge for Teaching

Teaching mathematics involves more than topics and procedures, however. Teaching also involves **using tools and skills for reasoning about mathematical ideas, representations, and solutions**, as well as knowing what constitutes coursework. It is **the "more" of more understanding of the insides of ideas, their roots and connections, their reasons and ways of being represented**. Second, **knowledge for teaching mathematics is different from the mathematical knowledge needed for other mathematically-intensive occupations and professions**. The mathematical problems and challenges of teaching are not the same as those faced by engineers, nurses, physicists, or astronauts. **Interpreting someone else's error, representing ideas in multiple forms, developing alternative explanations, choosing a usable definition – these are all examples of the problems that teachers must solve.** These are **genuine mathematical problems central to the work of teaching.** And, third, the mathematical knowledge needed for teaching must be usable for those mathematical problems. Mathematical knowledge for teaching must be serviceable for the mathematical work that teaching entails, **from offering clear explanations**, to **posing good problems to students**, to **mapping across alternative models**, to **examining instructional materials with a keen and critical mathematical eye**, to **modifying or correcting inaccurate or incorrect expositions**. **The mathematical knowledge needed for teaching, even at the elementary level, is not a watered-down version of "real" mathematics. Teaching mathematics is a serious and demanding arena of mathematical work.**

High-Leverage Practices

Deborah Ball (2000)

TeachingWorks
UNIVERSITY of MICHIGAN

2017

Great teachers aren't born.
THEY'RE TAUGHT.

1. Leading a group discussion
2. Explaining and modeling content, practices, and strategies
3. Eliciting and interpreting individual students' thinking
4. Diagnosing particular common patterns of student thinking and development in a subject-matter domain
5. Implementing norms and routines for classroom discourse and work
6. Coordinating and adjusting instruction during a lesson
7. Specifying and reinforcing productive student behavior
8. Implementing organizational routines
9. Setting up and managing small group work
10. Building respectful relationships with students
11. Talking about a student with parents or other caregivers
12. Learning about students' cultural, religious, family, intellectual, and personal experiences and resources for use in instruction
13. Setting long- and short-term learning goals for students
14. Designing single lessons and sequences of lessons
15. Checking student understanding during and at the conclusion of lessons
16. Selecting and designing formal assessments of student learning
17. Interpreting the results of student work, including routine assignments, quizzes, tests, projects, and standardized assessments
18. Providing oral and written feedback to students
19. Analyzing instruction for the purpose of improving it

Deborah Ball (2000)

TeachingWorks
UNIVERSITY of MICHIGAN

2017

Great teachers aren't born.
THEY'RE TAUGHT.

15. Checking student understanding during and at the conclusion of lessons

Teachers use a variety of informal but deliberate methods to assess what students are learning during and between lessons. These frequent checks provide information about students' current level of competence and help the teacher adjust instruction during a single lesson or from one lesson to the next. They may include, for example, simple questioning, short performance tasks, or journal or notebook entries.

Vicenç Font (2012)

El perfil del docente de Matemáticas. Una propuesta

Víctor Larios Osorio
Vicenç Font Moll
Patricia I. Spíndola Yáñez
Carmen Sosa Garza
Joaquín Giménez Rodr íguez

A) Competencias genéricas o transversales:

- Ciudadanía.
- Comunicación.
- Aprender a aprender.
- Competencia digital.

B) Competencias específicas o profesionales:

- Conocimiento del contenido matemático a enseñar.
- Conocimiento epistemología del contenido.
- Contextualización y valor interdisciplinar.
- Desarrollo del alumnado.
- Elementos socioculturales en la educación matemática.
- Análisis de contratos y normas matemáticas.
- Análisis y selección de contenidos.
- Diseños de evaluación.
- Análisis de secuencias.

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5.1. Conocimiento del contenido matemático

Es posible que el aspecto al que más se hace referencia sobre la formación del docente es el referente al conocimiento del área disciplinar. En efecto, hasta hace un par de décadas la visión predominante era que este aspecto era el necesario y (prácticamente) el suficiente para impartir clases de matemáticas en los niveles medio y superior. Con el paso del tiempo ha quedado en evidencia de que esta visión es más bien corta y que este aspecto no es suficiente para llevar a cabo una enseñanza adecuada. Esto no quiere decir no sea un aspecto necesario, sino que es indispensable y por ello se plantea en primer lugar de estas competencias:

El docente debe conocer y usar el contenido matemático a enseñar de manera suficientemente amplia, de modo que le permita realizar su función docente con seguridad y adaptarse a nuevos cambios curriculares si es necesario.

En un nivel básico se encuentra la posibilidad de que el profesor "acredite" los contenidos planteados en el currículo de los niveles educativos en que debe enseñar. No obstante, su labor requiere en segunda instancia de que domine los contenidos matemáticos del currículo del nivel que imparte incluyendo el saber resolver situaciones asociadas a dichos contenidos. Lo ideal sería que el docente profundice y amplíe los contenidos matemáticos más allá del currículo del nivel que imparte y sea consciente de la diferencia.

Es importante hacer hincapié en el hecho de que la expresión "conoce y usa" no implica un conocimiento mecánico que pueda aplicarse a ejercicios, sino que es una concepción más amplia que incluye el manejo de los objetos matemáticos (en el sentido de D'Amore, 2006b) y que incluye conceptos, procesos, lenguaje, etcétera) para llevar a cabo procesos amplios que incluyen la modelación, la validación, la experimentación, la exploración, etcétera. En el primer capítulo de este libro se ha presentado una idea al respecto de lo que se puede considerar como el pensamiento matemático y

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5.2. Elementos socioculturales en la enseñanza de las matemáticas

Las matemáticas se han nutrido y han sido influenciadas por el desarrollo histórico y filosófico de la humanidad.

Es por ello que el docente debe justificar y usar el valor formativo y sociocultural de las matemáticas y de su evolución histórica en la construcción de la actividad matemática, así como relacionarlo con las diferentes propuestas de enseñanza y aprendizaje.

Esto se considera en dos aspectos relacionados, pues uno tiene que ver con su papel como docente frente a grupo y el otro como un actor que gestiona y propone conocimientos y habilidades que se impartirán en el aula.

Así que por un lado debe conocer la evolución histórica de las matemáticas para mencionar anécdotas y presentar introducciones históricas de los conceptos nuevos para los alumnos. Mas en un nivel mayor debe fomentar en sus alumnos la comprensión de los problemas históricos cuya solución ha dado lugar a los distintos conceptos que aprenden.

Y por otro lado el docente debe ser capaz en un primer nivel de discutir o comunicar información matemática cuando sea relevante, así como resolver los problemas matemáticos que encuentre en la vida diaria o en el trabajo

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Víctor Larios Osorio
Vicenç Font Moll
Patricia I. Spíndola Yáñez
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Esta competencia hay que distinguirla de la relativa al conocimiento del contenido matemático, pues aunque la competencia que se menciona a continuación requiere la otra (y más), el énfasis está en la selección y organización del contenido considerando las condiciones y los recursos disponibles cambiando.

El docente debe planificar, aplicar y analizar diferentes selecciones y organizaciones del contenido, mediante el uso de materiales y recursos, así como desarrollos teórico-prácticos de la Educación Matemática para identificar los valores del currículo del nivel educativo en que imparte clase.

Por un lado se plantea la necesidad de que el profesor conozca y compare diferentes materiales, recursos, tecnologías, etcétera, y metodologías de enseñanza de las matemáticas de acuerdo a diferentes criterios para así describir sus fortalezas y debilidades. Posteriormente debe seleccionar la metodología de enseñanza más adecuada según el contexto y el curso correspondiente, pudiendo diseñar secuencias de enseñanza de acuerdo a la metodología seleccionada. Esto es con la finalidad de implementar dichas secuencias de una manera consciente.

Por otro lado, el profesor debe conocer las aportaciones teóricas de la didáctica de las matemáticas a cada uno de los ejes del currículo que le toca manejar. Además debe tener presentes estas aportaciones en las fases de diseño, implementación y evaluación de secuencias didácticas. Finalmente, un nivel más alto de la competencia es el conocer y usar los espacios de participación y comunicación de ideas profesionales sobre la didáctica de las matemáticas y sus diferentes enfoques.

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Esta competencia tiene que ver con la posibilidad de que el profesor eche mano de las competencias ya mencionadas para valorar posibles secuencias didácticas utilizando criterios específicos y así poder argumentar al respecto.

Esto se refiere a que el docente debe diseñar, aplicar y valorar secuencias de aprendizaje mediante técnicas de análisis didáctico y criterios de calidad para establecer ciclos de planificación, implementación, valoración y así plantear propuestas de mejora.

Esto, por un lado, incluye que el profesor muestre conocimiento del currículo de matemáticas como elemento fundamental para comprender su práctica pedagógica. Además es que sea capaz de integrar teorías, metodologías y currículo en la planificación de los procesos de enseñanza y reconoce las implicaciones en su práctica considerando los contextos institucionales. En un tercer nivel está el que implemente la planificación de los procesos de enseñanza en las prácticas y emita juicios argumentados y reflexivos acerca de las teorías, metodologías y el currículo.

Por otro lado incluye que aplique herramientas para describir las prácticas, objetos y procesos matemáticos presentes en un proceso de enseñanza-aprendizaje y muy en especial en su propia práctica. Además se requiere que conozca y aplique herramientas socioculturales para conocer la interacción y las normas que condicionan un proceso de enseñanza-aprendizaje. En un nivel mayor explica los fenómenos didácticos observados en los procesos de enseñanza-aprendizaje y muy en especial en su propia práctica.

Finalmente el profesor conoce criterios de calidad y los tiene presentes en la planificación de una secuencia didáctica de matemáticas. Además utiliza criterios de calidad para valorar procesos ya realizados de enseñanza

Llinares (2008)

CONSTRUIR EL CONOCIMIENTO NECESARIO PARA ENSEÑAR MATEMATICA: Practicas Sociales yTecnología

(adaptado por Enrique)

- Planificar y organizar el contenido matemático y las habilidades asociadas para ensenarlas determinando objetivos de aprendizaje, planes de acción y seleccionando definiciones y ejemplos apropiados para el público.
- Diseñar criterios de evaluación en coherencia con los objetivos de aprendizaje y valorar las producciones y respuestas de los estudiantes de acuerdo a estos criterios.
- Orientar el discurso matemático el en aula enfocado al aprendizaje de todos en función de los objetivos de aprendizaje.
- Formular preguntas que permitan vincular concepciones previas con lo nuevo, subrayar y valorar las diferentes aportaciones, y comprender el proceso mental de los estudiantes.
- Entender las dificultades y errores de los estudiantes, las razones de estas dificultades y tener la capacidad de predecir las posibles dificultades y de crear soluciones (incluyendo el desarrollo explicaciones alternativas y la representación ideas de múltiples formas) que permitan sobrepasar estas dificultades.
- Ser consciente del potencial de los diferentes instrumentos (técnicos y/o conceptuales) de los que dispone para realizar la actividad de enseñanza y ser capaz de elegirlos para usarlos adecuadamente

Harvard (2010)

Mathematical Quality of Instruction (MQI)

Center for Education Policy Research
HARVARD UNIVERSITY

The MQI is a Common Core-aligned observational rubric that provides a framework for analyzing mathematics instruction in several domains. Within each of the five domains, individual codes contain score points that categorize instruction into different levels of quality. The MQI was developed in order to provide a both multidimensional and balanced view of mathematics instruction.

The MQI instrument captures the nature of the mathematical content available to students during instruction, as expressed in teacher-student, teacher-content, and student-content interactions.

Each recorded lesson is divided into equal-length (e.g., 5 or 7.5 minute) segments for scoring. Two raters independently give each segment a score for each of these five MQI domains. Using short segments allows raters to capture events as they happen, without resorting to memory or notes at the end of the lesson. Raters also each give the whole lesson an overall MQI score as well as scores for other factors such as the pacing of the lesson, the density of mathematics in a lesson, and the extent to which the tasks and activities assigned develop mathematics.

Harvard (2010)

Mathematical Quality of Instruction (MQI)

Cinco dominios:

1. Common Core-Aligned Student Practices
2. Working with Students and Mathematics
3. Richness of Mathematics
4. Errors and Imprecision
5. Classroom Work is Connected to Mathematics

Harvard (2010)

Mathematical Quality of Instruction (MQI)

Common Core-Aligned Student Practices
This dimension captures the ways in which students engage with mathematical content. This includes:

- Whether **students ask questions and reason about mathematics** – e.g., students ask mathematically motivated questions, examine claims and counter-claims, or make conjectures.
- Whether students provide **mathematical explanations** spontaneously or upon request by the teacher.
- The **cognitive requirements** of a specific task – e.g., are students asked to find patterns, draw connections, determine the meaning of mathematical concepts, or explain and justify their conclusions.

Working with Students and Mathematics
This dimension captures whether teachers can "hear" and understand what students are saying, mathematically, and respond appropriately. Specifically:

- Whether the teacher accurately interprets and **responds to students' mathematical ideas**.
- Whether the teacher **remediates student errors** thoroughly, with attention to the specific misunderstandings that led to the errors.

Harvard (2010)

Mathematical Quality of Instruction (MQI)

Richness of Mathematics
Richness includes two elements: attention to the meaning of mathematical facts and procedures and engagement with mathematical practices and language.

- **Meaning-making** includes explanations of mathematical ideas and drawing connections among different mathematical ideas (e.g., fractions and ratios) or different representations of the same idea (e.g., number line, counters, and number sentence).
- **Mathematical practices** include the presence of multiple solution methods, where more credit is given for comparisons of solution methods for ease or efficiency; developing mathematical generalizations from specific examples; and the fluent and precise use of mathematical language.

Errors and Imprecision
This dimension refers to mathematical errors and distortion of content by the teacher. Specifically:

- Whether the teacher makes **content errors** that indicate gaps in the teacher's mathematical knowledge.
- Whether teacher talk features **imprecision in language and notation**, for instance when teachers cannot articulate mathematical ideas.
- Whether there is a **lack of clarity** in the presentation of content or the launch of tasks.

Classroom Work is Connected to Mathematics
This dimension captures whether classroom work has a mathematical point, or whether the bulk of instructional time is spent on **activities that do not develop mathematical ideas**—e.g. coloring, cutting and pasting—or non-productive uses of time such as transitions or discipline.

Harvard (2010)

Mathematical Quality of Instruction (MQI)

Parte de la rúbrica del dominio de errores e imprecisión

Errors and Imprecision		
This category is intended to capture teacher errors or imprecision in language and notation, uncorrected student errors, or the lack of clarity/precision in the teacher's presentation of the content. Do not code errors if these errors are captured and addressed within the segment or chapter (in this case, code as “low").		
Major Mathematical Errors or Serious Mathematical Oversights		
• Solving problems incorrectly • Defining terms incorrectly • Forgetting a key condition in a definition • Equating two non-identical mathematical terms, etc.		
Low	**Mid**	**High**
Instruction is *clean* of major errors in spoken or written work OR errors that occur are captured and corrected within the segment.	Teacher makes *major errors* either in spoken or written work or teacher neglects to discuss key aspects of a problem (e.g., forgetting a step, forgetting to finish the problem). The errors occur in ***part*** of the segment.	Teacher makes *major errors* either in spoken or written work or teacher neglects to discuss key aspects of a problem (e.g., forgetting a step, forgetting to finish the problem). The errors occur in ***most*** of the segment.

Harvard (2010)

Mathematical Quality of Instruction (MQI)

Parte de la rúbrica del dominio de errores e imprecisión

Imprecision in Language or Notation (mathematical symbols)		
• Errors in notation • Errors in mathematical language • Errors in general language		
Low	**Mid**	**High**
Instruction is *clean* of errors in mathematical language, general language and notation. Any errors made and quickly corrected should also be coded here.	Teacher makes a *small number* of momentary errors in notation, mathematical or general language.	Instruction is characterized of linguistic and notational *sloppiness across the segment* and/or by *major* notational and linguistic errors in even a small number of mathematical terms.

Clarification:

- ***Notation*** includes conventional mathematical symbols, such as +, -, =, or symbols for fractions and decimals, square roots, angle notation, functions, probabilities, exponents. Errors in notation might include inaccurate use of the equals sign, parentheses, or division symbol. By "conventional notation," we do not mean use of numerals or mathematical terms.
- ***Mathematical language*** includes technical mathematical terms, such as "angle," "equation," "perimeter," and "capacity." If a teacher uses these terms incorrectly, code as an error. When the focus is on a particular term or definition, also code errors in spelling or grammar.
- Teachers often use "***general language***" to convey mathematical concepts (i.e., explaining mathematical ideas or procedures in non-technical terms). General language also includes analogies, metaphors, and stories. Appropriate use of terms includes care in distinguishing everyday meanings different from their mathematical meanings. If teacher is unclear in his/her general talk about mathematical ideas, terms, concepts, procedures, code as an error.

Harvard (2010)

Mathematical Quality of Instruction (MQI)

Parte de la rúbrica del dominio de errores e imprecisión

Lack of Clarity		
• Teacher utterances cannot be understood, e.g.: • Mathematical point is muddled, confusing or distorted • Language or major errors make it difficult to discern the point • Teacher neglects to clearly solve the problem or explain content • Teachers' launch of a task/activity lacks clarity (the "launch" is the teacher's effort to get the mathematical tasks/activities in play)		
Low	**Mid**	**High**
Teacher's presentation of the mathematical content and/or launching of tasks is *clear* and *unambiguous.*	Teacher's presentation of the content and/or launching of tasks is *not clear* for *portions* of the segment.	Teacher's presentation of the mathematical content is *unclear*, *vague* or *incomplete* for most of the segment. Teacher's work is *muddled* or *confusing* and *severely distorts* the mathematical essence of the content. Also, teacher conveys mathematical tasks or problems *incompletely* or in a *confusing* manner.

Overall Errors and Imprecision		
Note: This is an overall code for each segment/chapter. It is not an average of the above, but an overall estimate of the errors and imprecision in instruction.		
Low	**Mid**	**High**
No errors occur. Do not use this code if "mid" or "high" is marked in any category above.	Brief error or errors generally not serious enough to indicate teacher may lack mathematical knowledge.	Either multiple small errors, **consistent lack of clarity** or one large error to suggest that teacher may lack key mathematical knowledge.

Harvard (2010)

Mathematical Quality of Instruction (MQI)

Parte de la rúbrica del dominio de participación de estudiantes, razonamiento y creación de sentido.

Overall Student Participation in Meaning-Making and Reasoning		
This code attempts to capture evidence of students' involvement in "doing" mathematics and the extent to which students participate in and contribute to meaning-making and reasoning. • During ***active instruction segments***, this mainly occurs through student mathematical statements: reasoning, explanations, question-asking. • During ***student work time***, this mainly occurs through work on a non-routine task. Note: This is an overall code for each segment/chapter. It is not an average of the above, but an overall estimate of the student participation in meaning-making and reasoning.		
Low	**Mid**	**High**
There are only a few or no examples of student participation in meaning making and reasoning. Tasks are largely procedural in nature. Students might also engage in unsystematic explorations.	Students engage with content at *mixed level*. Students may provide substantive explanations or ask a mathematically motivated question. May also include tasks with variable enactment (high and then low during segment).	Students contribute substantially to the building of mathematical ideas through the activities listed in this section as "high."