

Inductive lattices of totally composition formations

Retículos inductivos de formaciones totalmente compositivas

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ABSTRACT. Let τ be a subgroup functor such that all subgroups of every finite group G contained in $\tau(G)$ are subnormal in G . In this paper, we give a simple proof of the fact that the lattice of all τ -closed totally composition formations of finite groups is inductive.

Key words and phrases. Finite group, formation of groups, satellite of formation, τ -closed totally composition formation, inductive lattice of formations.

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RESUMEN. Sea τ un functor de subgrupo de modo que todos los subgrupos de cualquier grupo finito G contenido en $\tau(G)$ son subnormales en G . En este artículo, damos una demostración simple de que el retículo de todas las formaciones de composición totalmente τ -cerradas de los grupos finitos es inductivo.

Palabras y frases clave. Grupo finito, formación de grupos, satélite de formación, formación de composición totalmente τ -cerrada, retículo inductivo de formaciones.

1. Introduction

All groups considered in this paper are finite. A class of groups is a collection of groups satisfying the property that if a group G belongs to the collection, then every group isomorphic to G is also in the collection.

If a class of groups is a *formation*, it is closed with respect to forming quotient groups and subdirect products. This notion introduced by Gaschütz [3] in 1963 immediately became an object of extensive investigations. Saturated formations are very important in group theory; composition formations form a

broader family of formations. By Baer's theorem, composition formations are precisely solvably saturated formations [2, p. 373].

Skiba [10] introduced the concept of an inductive lattice of formations in order to adapt lattice-theoretical methods for the investigation of saturated formations. This concept plays an important role in the research of the lattices of formations and their law systems (see Chapter 4 of the book [10], Chapter 4 of the book [19]; and the papers [5, 6, 7, 8, 12, 13, 14, 18, 20, 21]).

Let Θ be a complete lattice of formations. A satellite f is called Θ -valued if all its values belong to Θ . We denote by Θ^c the set of all formations having a composition Θ -valued satellite. In [11, p. 901], it is shown that this set is a complete lattice of formations.

A complete lattice Θ^c is called *inductive* if for any collection of formations $\{\mathfrak{F}_i = CLF(f_i) \mid i \in I\}$, where f_i is an integrated satellite of $\mathfrak{F}_i \in \Theta^c$, the following equality holds:

$$\vee_{\Theta^c}(\mathfrak{F}_i \mid i \in I) = CLF(\vee_{\Theta}(f_i \mid i \in I)).$$

The inductance of a lattice Θ^c , in fact, means that a research of the operation \vee_{Θ^c} on the set Θ^c can be reduced to a research of the operation \vee_{Θ} on the set Θ . Therefore, the inductance is one very useful property of the lattice Θ^c .

Vorob'ev [17] proved that the lattice of all totally saturated formations is inductive. Moreover, it is already known that the lattice of all multiply composition formations is inductive (see [16]). However, the following question was still open.

Question. *Is the lattice of all totally composition formations inductive?*

The aim of the present paper is to give a simple proof of the following theorem which gives a positive answer to this question.

Theorem 1.1. *The lattice of all τ -closed totally composition formations c_{∞}^{τ} is inductive.*

2. Terminologies and notations

All unexplained notations and terminologies are standard. The reader is referred to [1, 2, 4, 11] if necessary.

2.1. Subgroup functor τ

In various applications of the theory of classes of finite groups, it is often necessary to use formations closed with respect to some subgroup systems. Skiba [10] introduced the concept of a subgroup functor, which covers all the systems of subgroups under consideration.

In each group G , we select a system of subgroups $\tau(G)$. We say that τ is a *subgroup functor* if (1) $G \in \tau(G)$ for every group G ; (2) for every epimorphism

$\varphi : A \rightarrow B$, and each $H \in \tau(A)$ and $T \in \tau(B)$, we have $H^\varphi \in \tau(B)$ and $T^{\varphi^{-1}} \in \tau(A)$.

If $\tau(G) = \{G\}$, then the functor τ is called *trivial*. A formation \mathfrak{F} is called τ -closed if $\tau(G) \subseteq \mathfrak{F}$ for every group G of \mathfrak{F} (see [10]).

We consider only subgroup functors τ such that for every group G all subgroups of $\tau(G)$ are subnormal in G .

2.2. Composition formations

The set of all primes is denoted by \mathbb{P} . Let $p \in \mathbb{P}$, and G a group. Then the subgroup $C^p(G)$ is the intersection of the centralizers of all the abelian p -chief factors of G , with $C^p(G) = G$ if G has no abelian p -chief factors.

For every collection of groups \mathfrak{X} , we write $\text{Com}(\mathfrak{X})$ to denote the class of all groups L such that L is isomorphic to some abelian composition factor of some group in \mathfrak{X} . If \mathfrak{X} is the set of one group G , then we write $\text{Com}(G)$ instead of $\text{Com}(\mathfrak{X})$.

The symbol $R(G)$ denotes the product of all solvable normal subgroups of G . We consider a function f of the form

$$f : \mathbb{P} \cup \{0\} \rightarrow \{\text{formations of groups}\}, \tag{*}$$

and the class of groups

$$CLF(f) = (G \mid G/R(G) \in f(0); G/C^p(G) \in f(p) \text{ for all } p \in \pi(\text{Com}(G))).$$

If \mathfrak{F} is a formation such that $\mathfrak{F} = CLF(f)$ for a function f of the form (*), then \mathfrak{F} is said to be *composition* (solvably saturated) formation, and f is said to be a *composition satellite* of \mathfrak{F} (see [4, p. 4]).

If the values of composition satellites of some formation are themselves composition formations, then this circumstance leads to the following natural definition. Every formation is 0-multiply composition; for $n > 0$, a formation \mathfrak{F} is called *n-multiply composition* if $\mathfrak{F} = CLF(f)$, and all nonempty values of f are $(n - 1)$ -multiply composition formations (see [11]).

A formation is called *totally composition* if it is n -multiply composition for all positive integers n .

2.3. Lattices of formations

A set of formations Θ is called a *complete lattice of formations* if the intersection of every set of formations in Θ belongs to Θ , and there is a formation \mathfrak{F} in Θ such that $\mathfrak{M} \subseteq \mathfrak{F}$ for every other formation \mathfrak{M} of Θ (see [10]).

Every complete lattice of formations is a complete lattice in the ordinary sense. Various collections of formations form complete lattices; for example, the set of all saturated formations [10, p. 151], and the set of all composition

(solvably saturated) formations [9, p. 97] are complete lattices of formations. Moreover for all positive integers n , the set of all n -multiply composition formations c_n , and the set of all totally composition formations $c_\infty = \bigcap_{n=1}^\infty c_n$ are complete lattices of formations (see [11, p. 904]).

A formation in Θ is called a Θ -formation. Let Θ be a complete lattice of formations, and let $\{\mathfrak{F}_i \mid i \in I\}$ be an arbitrary collection of Θ -formations. We denote

$$\vee_\Theta(\mathfrak{F}_i \mid i \in I) = \Theta\text{form}\left(\bigcup_{i \in I} \mathfrak{F}_i\right).$$

In particular, we write $\vee_\infty^\tau(\mathfrak{F}_i \mid i \in I) = c_\infty^\tau \text{form}(\cup_{i \in I} \mathfrak{F}_i)$.

If $\mathfrak{M}, \mathfrak{H} \in \Theta$, then $\mathfrak{M} \cap \mathfrak{H}$ is the greatest lower bound for $\{\mathfrak{M}, \mathfrak{H}\}$ in Θ ; and $\mathfrak{M} \vee_\Theta \mathfrak{H}$ is the least upper bound for $\{\mathfrak{M}, \mathfrak{H}\}$ in Θ .

Let $\{f_i \mid i \in I\}$ be a collection of Θ -valued functions of the form (*). Then by $\vee_\Theta(f_i \mid i \in I)$ we denote a function f such that

$$f(a) = \Theta\text{form}(\cup_{i \in I} f_i(a))$$

for all $a \in \mathbb{P} \cup \{0\}$.

3. Preliminaries

Following the paper [11], we set for every collection of groups \mathfrak{X} :

$$\mathfrak{X}(C^p) = \begin{cases} \text{form}(G/C^p(G) \mid G \in \mathfrak{X}) & \text{if } p \in \pi(\text{Com}(\mathfrak{X})); \\ \emptyset & \text{if } p \in \mathbb{P} \setminus \pi(\text{Com}(\mathfrak{X})). \end{cases}$$

We recall that the symbol \mathfrak{N}_p denotes the class of all p -groups. Let $\mathfrak{F} = CLF(F)$, where $F(0) = \mathfrak{F}$ and $F(p) = \mathfrak{N}_p \mathfrak{F}(C^p)$ for all $p \in \mathbb{P}$. Then the satellite F is called a *canonical composition satellite* of the formation \mathfrak{F} . By [11, Remark 1], every composition formation possesses a canonical composition satellite.

Lemma 3.1. [11, Lemma 8] *Let Θ be a complete lattice of formations such that $\Theta^c \subseteq \Theta$ and let the formation $\mathfrak{N}_p \mathfrak{H}$ belongs to Θ for each formation $\mathfrak{H} \in \Theta$ and every prime p . If $\mathfrak{F} = CLF(F) \in \Theta^c$, then the satellite F is Θ -valued.*

Lemma 3.2. [16, Lemma 1] *Let n be a positive integer. Then we have*

$$(c_{n-1}^\tau)^c = c_n^\tau.$$

Corollary 3.3. *The following equality holds: $(c_\infty^\tau)^c = c_\infty^\tau$.*

Proof. The inclusion $(c_\infty^\tau)^c \subseteq c_\infty^\tau$ is obvious. Let $\mathfrak{F} \in c_\infty^\tau$ and F be a canonical composition satellite of \mathfrak{F} . Then by Lemmas 3.1 and 3.2 for all $a \in \mathbb{P} \cup \{0\}$ and each positive integer n , the formation $F(a)$ is τ -closed n -multiply composition. Thus, the satellite F is c_∞^τ -valued. Consequently, $\mathfrak{F} \in (c_\infty^\tau)^c$, and we have $c_\infty^\tau \subseteq (c_\infty^\tau)^c$. □

Lemma 3.4. [22, Lemma 2.1] *Let $\mathfrak{F} = CLF(F)$ be a τ -closed n -multiply composition formation, where n is a positive integer. Then the satellite F is c_n^τ -valued.*

From Lemma 3.4 follows the corollary.

Corollary 3.5. *Let $\mathfrak{F} = CLF(F)$ be a τ -closed totally ω -composition formation. Then the satellite F is c_∞^τ -valued.*

Let $\{f_i \mid i \in I\}$ be a collection of composition satellites. Then by $\bigcap_{i \in I} f_i$, we denote the composition satellite f such that $f(a) = \bigcap_{i \in I} f_i(a)$ for all $a \in \mathbb{P} \cup \{0\}$ (see [11]).

Lemma 3.6. [11, Lemma 2] *Let $\mathfrak{F} = \bigcap_{i \in I} \mathfrak{F}_i$, where $\mathfrak{F}_i = CLF(f_i)$. Then $\mathfrak{F} = CLF(f)$, where $f = \bigcap_{i \in I} f_i$.*

Let $\{f_i \mid i \in I\}$ be the collection of all composition c_∞^τ -valued satellites of a formation \mathfrak{F} . Since the lattice c_∞^τ is complete using Lemma 3.6, we conclude that $f = \bigcap_{i \in I} f_i$ is a composition c_∞^τ -valued satellite of \mathfrak{F} . The satellite f is called *minimal*.

Let Θ be a complete lattice of formations. Then $\Theta\text{form}\mathfrak{X}$ is the intersection of all Θ -formations containing a collection of groups \mathfrak{X} . Thus, $c_\infty^\tau\text{form}\mathfrak{X}$ is the intersection of all τ -closed totally composition formations containing a collection of groups \mathfrak{X} . The next lemma immediately follows from [11, Lemma 5] by Corollary 3.3, and gives a description of the minimal c_∞^τ -valued satellite of a formation $c_\infty^\tau\text{form}\mathfrak{X}$.

Lemma 3.7. *Let \mathfrak{X} be a nonempty collection of groups, $\mathfrak{F} = c_\infty^\tau\text{form}\mathfrak{X}$, $\pi = \pi(\text{Com}(\mathfrak{X}))$, and let f be the minimal c_∞^τ -valued composition satellite of \mathfrak{F} . Then the following statements hold:*

- 1) $f(0) = c_\infty^\tau\text{form}(G/R(G) \mid G \in \mathfrak{X})$;
- 2) $f(p) = c_\infty^\tau\text{form}(G/C^p(G) \mid G \in \mathfrak{X})$ for all $p \in \pi$;
- 3) $f(p) = \emptyset$ for all $p \in \mathbb{P} \setminus \pi$;
- 4) if $\mathfrak{F} = CLF(h)$ and the satellite h is c_∞^τ -valued, then for all $p \in \pi$ we have

$$f(p) = c_\infty^\tau\text{form}(G \mid G \in h(p) \cap \mathfrak{F} \text{ and } O_p(G) = 1), \text{ and}$$

$$f(0) = c_\infty^\tau\text{form}(G \mid G \in h(0) \cap \mathfrak{F} \text{ and } R(G) = 1).$$

By Lemma 3.7, it is easy to show the following assertion.

Corollary 3.8. *Let f_1 and f_2 be the minimal composition c_∞^τ -valued satellites of formations \mathfrak{F}_1 and \mathfrak{F}_2 respectively. Then $\mathfrak{F}_1 \subseteq \mathfrak{F}_2$ if and only if $f_1 \leq f_2$.*

If $\mathfrak{F} = CLF(f)$ and $f(a) \subseteq \mathfrak{F}$ for all $a \in \mathbb{P} \cup \{0\}$, then f is called an *integrated* satellite of \mathfrak{F} .

4. Inductance of the lattice $c_{\omega_\infty}^\tau$

Proof of Theorem. Let $\{\mathfrak{F}_i \mid i \in I\}$ be a collection of τ -closed totally composition formations, and f_i be an integrated c_∞^τ -valued composition satellite of \mathfrak{F}_i . Let

$$\mathfrak{F} = CLF(f) = \vee_\infty^\tau(\mathfrak{F}_i \mid i \in I), \text{ and } \mathfrak{M} = CLF(\vee_\infty^\tau(f_i \mid i \in I)).$$

We shall show that $\mathfrak{F} = \mathfrak{M}$ proceeding by induction on i .

Step 1. Let $i = 2$, $p \in \mathbb{P}$, and h_j be the minimal c_∞^τ -valued composition satellite of the formation $\mathfrak{F}_j = CLF(f_j)$, where $j = 1, 2$. Then by Corollary 3.5, we have

$$h_j(p) \subseteq f_j(p) \subseteq \mathfrak{N}_p h_j(p) = F_j(p) \in c_\infty^\tau,$$

where F_j is the canonical c_∞^τ -valued composition satellite of the formation \mathfrak{F}_j . Let $\mathfrak{F} = CLF(F)$, where F is the canonical c_∞^τ -valued composition satellite of the formation \mathfrak{F} . Then by Lemma 3.7, we have

$$\begin{aligned} h(p) &= c_\infty^\tau \text{form}((\mathfrak{F}_1 \cup \mathfrak{F}_2)(C^p)) = c_\infty^\tau \text{form}(\mathfrak{F}_1(C^p) \cup \mathfrak{F}_2(C^p)) = \\ &= c_\infty^\tau \text{form}(h_1(p) \cup h_2(p)) \subseteq f(p) \subseteq \\ &= \mathfrak{N}_p c_\infty^\tau \text{form}(h_1(p) \cup h_2(p)) = \mathfrak{N}_p h(p) = F(p). \end{aligned}$$

Thus, we have $h(p) \subseteq f(p) \subseteq F(p)$ for all $p \in \mathbb{P}$; moreover, it holds $h(0) \subseteq f(0) \subseteq F(0)$. Hence, $h(a) \subseteq f(a) \subseteq F(a)$ for all $a \in \mathbb{P} \cup \{0\}$ implies $h \leq f \leq F$. Consequently, we have $\mathfrak{F}_1 \vee_\infty^\tau \mathfrak{F}_2 = CLF(f_1 \vee_\infty^\tau f_2)$.

Step 2. Let $i > 2$, and the assertion is true for $i = r - 1$ by induction. Then $\mathfrak{F}_1 \vee_\infty^\tau \dots \vee_\infty^\tau \mathfrak{F}_{r-1} = CLF(f_1 \vee_\infty^\tau \dots \vee_\infty^\tau f_{r-1})$. By Step 1, we have

$$\mathfrak{F} = c_\infty^\tau \text{form}((\mathfrak{F}_1 \vee_\infty^\tau \dots \vee_\infty^\tau \mathfrak{F}_{r-1}) \cup \mathfrak{F}_r) = CLF(f),$$

where

$$\begin{aligned} f(a) &= c_\infty^\tau \text{form}((f_1(a) \vee_\infty^\tau \dots \vee_\infty^\tau f_{r-1}(a)) \cup f_r(a)) = \\ &= f_1(a) \vee_\infty^\tau \dots \vee_\infty^\tau f_r(a) = (f_1 \vee_\infty^\tau \dots \vee_\infty^\tau f_r)(a) \end{aligned}$$

for each $a \in \mathbb{P} \cup \{0\}$. Therefore, we have $f = f_1 \vee_\infty^\tau \dots \vee_\infty^\tau f_r$. This proves the theorem. \square

Each complete sublattice of the inductive lattice is an inductive lattice. Thus, we have the following result.

Corollary 4.1. *Let θ be a complete sublattice of the lattice c_∞^τ . Then θ is inductive.*

If τ is trivial, we have the following result.

Corollary 4.2. *The lattice of all totally composition formations is inductive.*

Corollary 4.3. *The lattice of all solvable totally composition formations is inductive.*

By Lemma 3.6, we have the following corollary.

Corollary 4.4. *Let $\xi(x_1, \dots, x_m)$ be a term of signature $\{\cap, \vee_\infty^\tau\}$, and let f_i be an integrated c_∞^τ -valued composition satellite of a formation \mathfrak{F}_i , where $i = 1, \dots, m$. Then, we have $\xi(\mathfrak{F}_1, \dots, \mathfrak{F}_m) = CLF(\xi(f_1, \dots, f_m))$.*

5. Some applications

Let A be a group, and p be a prime. We use $Z_p \wr A$ to denote the regular wreath product of groups Z_p and A (see [2, p. 66]).

Lemma 5.1. *Let $\mathfrak{F}_i = c_\infty^\tau \text{form}(Z_p \wr A_i)$, where $p \notin \pi(A_i)$ for $i = 1, 2$. Then $f(p) = f_1(p) \cap f_2(p)$, where f_i and f are the minimal c_∞^τ -valued composition satellites of the formations \mathfrak{F}_i and $\mathfrak{F} = \mathfrak{F}_1 \cap \mathfrak{F}_2$, respectively.*

Proof. See proof of [15, Lemma 3.1]. □

The following lemma is proved by direct calculation.

Lemma 5.2. *Let f_i be the minimal c_∞^τ -valued composition satellite of a formation \mathfrak{F}_i , where $i \in I$. Then $\vee_\infty^\tau(f_i \mid i \in I)$ is the minimal c_∞^τ -valued composition satellite of the formation $\mathfrak{F} = \vee_\infty^\tau(\mathfrak{F}_i \mid i \in I)$.*

Proposition 5.3. *Let $\mathfrak{F}_i = c_\infty^\tau \text{form}(Z_p \wr A_i)$ for $p \notin \pi(A_i)$, where $i = 1, \dots, m$. Let f_i be the minimal c_∞^τ -valued composition satellite of \mathfrak{F}_i and $f(p) = \xi(f_1, \dots, f_m)(p)$, where $\xi(x_1, \dots, x_m)$ is a term of signature $\{\cap, \vee_\infty^\tau\}$. Then f is the minimal c_∞^τ -valued composition satellite of the formation $\mathfrak{F} = \xi(\mathfrak{F}_1, \dots, \mathfrak{F}_m)$.*

Proof. Let $h = \xi(f_1, \dots, f_m)$. By Corollary 4.4, we have

$$\xi(\mathfrak{F}_1, \dots, \mathfrak{F}_m) = CLF(h).$$

We shall show that $h(p) = f(p)$ by induction on the number r of occurrences of the symbols in $\{\cap, \vee_\infty^\tau\}$ into ξ .

The case $r = 1$ holds using Lemmas 5.1 and 5.2.

Let the term ξ have $r > 1$ occurrences of the symbols in $\{\cap, \vee_\infty^\tau\}$. Let ξ have the form $\xi(x_1, \dots, x_m) = \xi_1(x_{i_1}, \dots, x_{i_a}) \Delta \xi_2(x_{j_1}, \dots, x_{j_b})$, where $\{x_{i_1}, \dots, x_{i_a}\} \cup \{x_{j_1}, \dots, x_{j_b}\} = \{x_1, \dots, x_m\}$, and $\Delta \in \{\cap, \vee_\infty^\tau\}$. We suppose that the assertion is true for the terms ξ_1 and ξ_2 . By induction, we have $h_1(p) = \xi_1(f_{i_1}, \dots, f_{i_a})(p)$ and $h_2(p) = \xi_2(f_{j_1}, \dots, f_{j_b})(p)$, where h_1 and h_2 are the minimal c_∞^τ -valued composition satellites of the formations $\xi_1(\mathfrak{F}_{i_1}, \dots, \mathfrak{F}_{i_a})$ and $\xi_2(\mathfrak{F}_{j_1}, \dots, \mathfrak{F}_{j_b})$, respectively. Thus, we have

$$f(p) = h_1(p) \Delta h_2(p) =$$

$$\begin{aligned} \xi_1(f_{i_1}(p), \dots, f_{i_a}(p)) \triangle \xi_2(f_{j_1}(p), \dots, f_{j_b}(p)) = \\ \xi(f_1(p), \dots, f_m(p)) = \xi(f_1, \dots, f_m)(p) = h(p), \end{aligned}$$

as claimed. ☑

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